Ion Pump Based Bio-Synthetic Modulator Model for Diffusive Molecular Communications

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Abstract—In diffusive molecular communication (DMC), the transmitter has to be able to control the release of signaling molecules for modulation of the information bits. In natural cells, pumping ions is an important control mechanism for releasing molecules which is carried out by ion pumps embedded in the membrane. The activity of the ion pumps is controlled by a driving parameter. In particular, light driven pumps are controlled by light intensity and enable a high degree of spatial and temporal control for modulation functionality. In this paper, a modulator based on ion pumps is proposed for DMC which controls the release rate of the molecules from the transmitter by modulating a light intensity signal. The pumping process of the ion pump is modeled by a Markov model based on which the stochastic nature of the modulated signal, i.e., the release rate of the ions from the transmitter is analyzed. A simple on-off keying modulation scheme is realized based on the proposed modulator. Our numerical results show that a realistic transmitter can not release ions instantaneously nor deterministically.

I. INTRODUCTION

Diffusion-based molecular communication (DMC) among nano-machines is expected to have many applications in various areas including the healthcare, industrial, and environmental sectors [1]. To implement nano-communication networks, it is convenient to exploit mechanisms already existing in natural biological systems for the modeling and design of the functionalities required in DMC systems [2], [3]. In particular, signal modulation at the transmitter nano-machine is an important functionality where the transmitter has to be able to control the release of the molecules.

In the existing DMC literature, the transmitter nano-machine is typically assumed to have at least one of the following two ideal properties. First, the transmitter is assumed to release molecules instantaneously at the beginning of a symbol interval; second, the transmitter is assumed to be able to release any desired number of molecules deterministically without any uncertainty [4]–[8]. However, in a real system, the transmitter will be a biological or electronic nano-machine (e.g. a modified cell) which controls the release of the information molecules into the channel using e.g. electrical, chemical, or optical signals [9]. Because of non-zero time constants and the inherent randomness of any release processes, the molecules will not enter the channel instantaneously and their number will not be deterministic. The authors in [10] assume the transmitter is able to maintain any desired concentration gradient between the inside and the outside of the transmitter. Furthermore, the author in [11] assumes different chemical reactions generate the emission patterns of different symbols. However, the physical characteristics of the release control mechanism such as the transmitter geometry and forces caused by concentration gradients or pumping mechanisms are not included in the models in [10], [11]. In [12], the authors model the transmitter as a spherical structure whose surface is covered by nano-pores whose effect on the released molecules is modeled in a simplified manner via a modified diffusion coefficient. By mimicking the moth pheromone system, a bio-synthetic MC system has been recently proposed in [13] where temperature regulation controls the timely and precise release of the pheromones. In [14], a modulator model for DMC is proposed based on ion channels, which is one of the important membrane proteins that natural cells employ to control the release of ions. Another important class of transport mechanisms in biological cells are ion pumps (ion transporters) [9, Chapter 12]. In contrast to the passive ion transport by ion channels, ion pumps are membrane proteins which actively transport ions across the cell membrane against the concentration gradient. Named based on the source of energy driving the ion pump, the two main types of ion pumps found in nature are light driven pumps and adenosine triphosphate (ATP) driven pumps [9, Chapter 12], see Fig. 1. Since light introduces a high degree of spatial and temporal flexibility and control, light driven pumps are highly attractive for implementing the modulation process in nano-machines.

In this paper, the transmitter is modeled as a spherical synthetic cell whose membrane is uniformly covered by light driven ion pumps. Nevertheless, the obtained results can be extended to ATP driven ion pumps. We propose a new modulator for DMC which we refer to as ion pump based bio-synthetic modulator (IPM). An IPM controls the release rate of the ions from the cell by modulating a light intensity signal. Thereby, the rate at which the ions are released from the cell constitutes the modulated signal.

To analyze the properties of IPMs, we consider a simple on-off keying modulation scheme where the on and off states
are controlled by applying different light intensities. More in detail, to transmit bit 0 in a given time slot, the light source is switched off, and to transmit bit 1, light with a given intensity is shined onto the cell membrane to pump the ions out of the cell. The pumping process of an ion pump for a given light intensity is modeled as a Markov process. Based on the Markov model, the release process of the ions is analyzed. By taking into account the contribution of all ion pumps embedded in the cell membrane, the statistics of the modulated signal of the IPM are characterized.

II. ION PUMP BASED BIO-SYNTHETIC MODULATOR

In this section, we introduce light driven pumps and model their functionality mathematically, describe the IPM transmitter exploiting ion pumps, and present an on-off keying modulation scheme based on the IPM transmitter.

A. Light Driven Pumps

Among the known light driven pumps, the Bacteriorhodopsin is the best-characterized light driven proton pump [16]. A corresponding simple model which includes the most essential properties of an active proton pump is given in [17]. Therefore, for simplicity of presentation, we adopt the Bacteriorhodopsin protein model in this paper. Nevertheless, the results can be easily extended to other types of light driven pumps.

In light driven pumps, after a photon is absorbed, several reactions accompanying configurational changes will occur in a sequential manner in the protein pump which ultimately lead to the transport of an ion of a specific type from the intracellular to the extracellular environment. The photocycle of Bacteriorhodopsin is depicted in Fig. 2. The photocycle starts in the resting state (BR), in which the protein is ready to absorb a photon [17], and then by passing the intermediate conformers, K, L, M₁, M₂, N, and O, the protein returns back to the resting state, BR. In the transition from M₁ to M₂ a proton \(^2\) \(H^+\) is pumped which constitutes the signaling molecule.

The photocycle after the absorption of a photon is modeled as a continuous time Markov process and described by the master equation in [17]\(^3\)

\[
\frac{dp_i(t)}{dt} = \sum_{j=1}^{S} R_{ij} p_j(t),
\]

in which \(p_j(t)\) is the probability that the system is in (conformer) state \(j\) at time \(t\), \(S\) is the number of the states, \(R_{ij}\) is the transition rate from state \(i\) to state \(j\), and \(R_{ii} = -\sum_{j\neq i} R_{ji}\). Writing (1) in matrix form results in

\[
\frac{dp(t)}{dt} = Rp(t),
\]

where \(p(t)\) is the vector of the state probabilities at time \(t\), and \(R\) is the transition rate matrix. Since matrix \(R\) has \(S - 1\) nonzero eigenvalues and one zero eigenvalue, solving (2) yields [17]

\[
p_i(t) = p_i^{eq} + \sum_{j=1}^{S-1} c_j e^{\mu_j t},
\]

where \(\mu_j, 1 \leq j \leq S - 1\) are the nonzero negative eigenvalues of matrix \(R\), \(E_{ij}\) is the \((i,j)\)th element of matrix \(E\) whose \(j\)th column is the eigenvector corresponding to eigenvalue \(\mu_j\), \(p_i^{eq}\) denotes the equilibrium probability of state \(i\) which is the \(i\)th element of vector \(p^{eq}\), the eigenvector corresponding to the zero eigenvalue, and \(c_j, 1 \leq j \leq S\), are constants that need to be adjusted to satisfy the initial conditions of (2). After absorption of the photon, the system is in state \(K\) (state 1), i.e., the initial conditions for (2) are \(p_1(0) = 1\) and \(p_i(0) = 0, 2 \leq i \leq S\). The final state of the photocycle (5th state) is the resting state, \(BR\), in which the protein is ready to absorb a new photon. The absorption process of a new photon depends on the light intensity in terms of the number of photons per unit time and unit area and the absorption cross-section of a protein in terms of unit area per molecule (protein) [19].

B. IPM Transmitter Model

We model the IPM transmitter as a spherical cell of radius \(r_{tx}\) covered by a membrane, see Fig. 3. The pump proteins are embedded in the membrane using biological engineering approaches [15]. The specific type of ions transported by the embedded pumps are referred to as type \(A\) ions (molecules) and constitutes the information molecules. In particular, Bacteriorhodopsin transports hydrogen ions (protons), \(H^+\). We assume \(N_p\) pumps are uniformly distributed over the cell membrane. The radius of each pump is assumed to be \(r_p\). The space is divided into an intracellular and an extracellular environment by the infinitely thin membrane. To control the release of the ions, the light intensity denoted by \(L(t)\), is utilized. For generation of the ions, we adopt a simple model where the ions are produced by an organelle inside the cell via some chemical process. The concentration of the ions inside the cell remains constant in the equilibrium state of the reaction. Based on the photocycle of Bacteriorhodopsin explained above, protons from the intracellular environment are transported to the extracellular environment. The proton transportation in the adopted model of the light driven pump is a function of the concentration of the protons inside and

\(^2\)The terms “proton”, “ion”, and “molecule” are used interchangeably in the remainder of the paper.

\(^3\)The Markov model in [17] uses a more accurate model of the protein pump which includes the protonation sites at each conformer as the states.
outside the cell which is reflected by two transition rates in the Markov model (transition from $M_1$ to $M_2$ and from $N$ to $O$). Taking into account the concentration variations during the pumping process makes the analysis cumbersome. Therefore, we assume that the concentration of the protons inside and outside the cell remains in the range which is optimal for pump operation. Assuming a sufficiently fast molecule generator, the released molecules are always replenished, and therefore assuming a constant concentration inside the cell is justified. For the concentration of the protons outside the cell, the concentration around the pumps is shown to remain in the optimal range in the simulation results section.

C. IPM Based On-off Keying Modulation

We propose a simple on-off keying modulation format for the proposed IPM. Assume time is divided into slots of length $T$. Bits 0 and 1 are represented by not releasing and releasing ions at the transmitter, respectively. For transmission of bit 0, the pumps must be kept in the resting state by not shining light on them, i.e., the light intensity for modulating bit 0 in time slot $[0,T]$ is $L_0(t) = 0$. In contrast, for transmission of bit 1, the pumps are exited by shining light with a nonzero intensity, $L_1 > 0$, of wavelength $\lambda$ on them to pump the ions outside the cell for a certain duration $T_1$ where $0 < T_1 < T$, and the pumps are kept in the resting state by turning the light off during the remainder of the time slot, i.e., for $T_2 = T - T_1$ seconds. Therefore, the light intensity function for modulating bit 1 in time slot $[0,T]$ is $L_1(t) = L_1(u(t) - u(t - T_1))$, where $u(t)$ is the unit step function.

We note that not releasing ions during the last $T_2$ seconds of the time slot for transmission of bit 1 allows cleansing of the channel from the previously released ions, if $T_2$ exceeds the length of the diffusion channel memory. Thereby, intersymbol interference (ISI) can be reduced or even avoided at the cost of a reduced symbol rate.

III. MODULATED SIGNAL FOR IPM

In this section, we propose a Markov model for the release process of a light driven pump which also includes the light absorption process. Furthermore, the statistical properties of the modulated signal are analyzed by utilizing the proposed Markov model.

A. Markov Model for IPM

We model the entire process of proton pumping, which also includes the photon absorption, by a two-state Markov model. State 0 in Fig. 4 denotes the resting state of the pump where it is ready to absorb a photon. By absorption of one photon, the system transitions to state 1 (in Fig. 4) where the pump photocycle starts. In this state, one ion is released and another ion is taken from the intracellular environment and stored in the pump for future release, and the system returns to the resting state (state 0) in which the protein is ready for the next photon absorption.

Assume the light is shined onto the pump with intensity $L_\lambda$ photon/(m$^2$s). The process of emission of photons per unit area is a Poisson process with parameter $L_\lambda$ [20]. The absorption cross-section is the effective area of the protein that a photon requires to be absorbed and is denoted by $\sigma_\lambda$ (m$^2$)/protein [21]. Given $\sigma_\lambda$, the process of photon absorption is a Poisson process with parameter $L_\lambda \sigma_\lambda$, because of the thinning property of the Poisson distribution. Therefore, given the pump is in the resting state at $t = 0$, the random variable for the photon absorption time denoted by $X_{01}$, is exponentially distributed with parameter $\gamma = L_\lambda \sigma_\lambda$, i.e., the probability density function (pdf) of $X_{01}$ is

$$P_{01}(t) = \gamma e^{-\gamma t}. \quad (4)$$

If the direction of the light is not perpendicular to the pump surface, the effective cross-section area is a function of the angle deviation. For instance, in the schematic transmitter in Fig. 3, if the pump is located at polar angle $\theta$, the effective absorption cross-section is $\sigma_\lambda^{\theta} = \sigma_\lambda \cos(\theta)$.

If the system is in state 1 at time $t = 0$, a photon has been absorbed. Given the absorption of a photon, the random variable representing the return time to the resting state is denoted by $X_{10}$ and characterized by pdf $P_{10}(t)$. Given the absorption of a photon, the probability of finding the system in the resting state (state number $S$) at time $t$ is obtained from (3) as follows,

$$p_S(t) = p^S \sum_{j=1}^{S-1} c_j E_{Sj} e^{\mu_j t}. \quad (5)$$

Since the system does not return to the previous state in the resting state and waits for the absorption of the next photon, it is deduced that $P_{10}(t)$ is $p_{S}(t)$. Therefore, we have

$$P_{10}(t) = \frac{dp_S(t)}{dt} = \sum_{j=1}^{S-1} c_j E_{Sj} \mu_j e^{\mu_j t}. \quad (6)$$

For the case $S = 2$, (6) is an exponential distribution and then the transition process from state 1 to 0 is a Poisson process which is memoryless. In particular, the entire process would be the well-known birth and death Markov process. Nevertheless, for *Bacteriorhodopsin* we have $S > 2$ and (6) is an affine combination of $S - 1$ exponential functions $\mu_j e^{\mu_j t}$, $j = 1, \cdots, S - 1$, which leads to a distribution function with memory.

B. Modulated Signal

Assuming the transmission of bit 1 in time slot $[0,T]$, the light intensity function is $L_1(t) = L_\lambda (u(t) - u(t - T_1))$. Consider pump $g \in \{1, \cdots, N_p\}$, which is located at polar angle $0 \leq \theta_g \leq \frac{\pi}{2}$. We analyze the ion release process of this pump during time $[0,T_1]$ given light intensity $L_\lambda$. From the proposed Markov model, it is deduced that pumping one ion is equivalent to one round trip of the pump system between...
where the Laplace transform of (10), we obtain i.e., the absorption process is independent of the subsequent

\[ P_{01} = \sum_{j=1}^{S-1} \alpha_j e^{\theta_j \gamma} + \sum_{j=1}^{S} \alpha_j \mu_j e^{\theta_j \gamma} \]

where * is the convolution operator, \( \alpha_j = \frac{\gamma(\theta_j)c_j E_{Sj}}{\gamma(\theta_j) + \mu_j} \) for 1 \( \leq j \leq S \), \( \mu_S = -\gamma(\theta_j) \), and

\[ \alpha_S(\theta_q) = \sum_{j=1}^{S} c_j E_{Sj} \mu_j. \]

Since \( \int_{0}^{\infty} P_X(t) dt = 1 \), we have \( \sum_{j=1}^{S} \alpha_j = 1 \). Therefore, \( P_X(t) \) is an affine combination of \( S \) exponential functions \( -\mu_j e^{\theta_j \gamma} \), \( j = 1, \ldots, S \).

Let random variable \( X_q \) denote the duration of the \( i \)th round trip or equivalently the \( i \)th ion released by pump \( q \). Then, the \( n \)th round trip of the system (equivalent to the release of the \( n \)th ion) occurs at time

\[ Y_q^n = \sum_{i=1}^{n} X_q^i. \]

Since the \( X_q^i, i = 1, \ldots, n, \) are mutually independent, the pdf of \( Y_q^n \), \( P_{Y_q^n}(t) \), is given by

\[ P_{Y_q^n}(t) = P_{X_q^1}(t) \ast \cdots \ast P_{X_q^n}(t), \]

where the \( P_{X_q^i}(t), i = 1, \ldots, n, \) are given by (7). Taking the Laplace transform of (10), we obtain

\[ \overline{P_{Y_q^n}}(s) = \prod_{i=1}^{n} P_{X_q^i}(s) = \left[ \sum_{j=1}^{S} \alpha_j e^{\theta_j \gamma} \right]^{n} \]

where \( \overline{P(\cdot)} \) denotes the Laplace transform of \( P(\cdot) \).

Now, we define random variable \( N_q(t) \) as the number of the round trips of the pump \( q \) (the number of the released ions) until time \( t \). It is easy to see that the event \( \{ N_q(t) \geq n \} \) is equivalent to the event \( \{ Y_q^n \leq t \} \), i.e.,

\[ \{ N_q(t) \geq n \} = \{ Y_q^n \leq t \}. \]

Consider the small time interval \( [t, t + \Delta t] \subset [0, T_1] \). The number of ions released by pump \( q \) in this interval is \( N_q(t + \Delta t) - N_q(t) \). The duration \( \Delta t \) is chosen sufficiently small such that at most one ion is released in this interval. The probability that \( N_q(t + \Delta t) - N_q(t) = 1 \) is obtained based on the fact that one ion may be released during the time interval \( [t, t + \Delta t] \) by the \( k \)th, \( k \geq 1 \), round trip of pump \( q \), i.e.,

\[ \Pr\{ N_q(t + \Delta t) - N_q(t) = 1 \} = \sum_{k=1}^{\infty} \Pr\{ t \leq Y_k^q \leq t + \Delta t \} = \sum_{k=1}^{\infty} P_{Y_k^q}(t) \Delta t. \]

As a result, \( N_q(t + \Delta t) - N_q(t) \) is a Bernoulli random variable with success probability given by (13). We define the random point process \( N(t) \) as

\[ N(t) = \sum_{q=1}^{N_p} N_q(t), \]

which counts the total number of round trips of all \( N_p \) pumps, i.e., the total number of ions released by all the pumps until time \( t \). Therefore, the number of ions released by all pumps in interval \( [t, t + \Delta t] \subset [0, T_1] \) is

\[ N(t + \Delta t) - N(t) = \sum_{q=1}^{N_p} (N_q(t + \Delta t) - N_q(t)), \]

where \( N_q(t + \Delta t) - N_q(t) \) is a Bernoulli random variable with success probability given by (13). We note that for disjoint intervals \( [t_1, t_1 + \Delta t] \) and \( [t_2, t_2 + \Delta t] \), the Bernoulli random variables \( N_q(t_1 + \Delta t) - N_q(t_1) \) and \( N_q(t_2 + \Delta t) - N_q(t_2) \) are independent, since the success probabilities \( \sum_{k=1}^{\infty} P_{Y_k^q}(t) \Delta t \) and \( \sum_{k=1}^{\infty} P_{Y_k^q}(t) \Delta t \) are very small. On the other hand, the Hodges-Le Cam Poisson approximation [22] states that the summation of independent Bernoulli random variables can be approximated by a Poisson distributed random variable, if the maximum success probability of the individual Bernoulli random variables is small. Choosing sufficiently small \( \Delta t \), the success probability in (13) is small and hence \( N(t + \Delta t) - N(t) \) is approximated as a Poisson random variable with mean

\[ \mathbb{E}\{ N(t + \Delta t) - N(t) \} = \sum_{q=1}^{N_p} \sum_{k=1}^{\infty} P_{Y_k^q}(t) \Delta t. \]

As a result, the process \( N(t) \) is a non-homogeneous Poisson point process with mean [23]

\[ \mathbb{E}\{ N(t) \} = \sum_{q=1}^{N_p} \sum_{k=1}^{\infty} P_{Y_k^q}(t). \]

The release rate of the ions from the cell, which constitutes the modulated signal, is given by

\[ w(t) = \lim_{\Delta t \to 0} \frac{N(t + \Delta t) - N(t)}{\Delta t}. \]

Therefore, \( w(t) \Delta t \) is Poisson distributed with mean

\[ \mathbb{E}\{ w(t) \} \Delta t = \sum_{q=1}^{N_p} \sum_{k=1}^{\infty} P_{Y_k^q}(t) \Delta t, \]

for small \( \Delta t \), where \( P_{Y_k^q}(t) \) is the inverse Laplace transform of (11).
IV. NUMERICAL AND SIMULATION RESULTS

Simulation Parameters: For the numerical and simulation results, a spherical transmitter of radius \( r_{tx} = 5 \) \( \mu m \) is considered with \( N_p = 100 \) Bacteriorhodopsin ion pumps of radius \( r_p = 2 \) nm (modeled as described in Section II.A) uniformly distributed over its surface. The values of the transition rates between the states of the Bacteriorhodopsin are taken from [18]. The rates in [18] have been obtained from experiments in PH= 7 and a temperature of 20.0 \( \degree C \), and are given in the Table I, where \( \kappa_f \) and \( \kappa_b \) denote the forward and backward transition rates, respectively. Light with intensity \( \mathcal{L}_\lambda \) is shined in the direction of \( -\hat{a}_z \) where \( \hat{a}_z \) is the unit vector in the direction of \( z \) axis in the Cartesian coordinate system, see Fig. 3.

Particle Based Simulator (PBS): In our PBS, for transmission of bit 1, the number of protons produced by pump \( q \) in an interval \( [t, t + \Delta t] \) is generated from a Poisson distribution with the mean given in (16) and the generated protons are released at the location of pump \( q \). The locations of the protons released by the transmitter are known. The protons perform independent random walks in the 3-dimensional space. The random walk of each molecule during \( \Delta t \) seconds in each dimension is modeled as a Gaussian random variable with zero mean and variance \( 2D\Delta t \) where \( D \) is diffusion coefficient of the environment and is assumed to be \( 10^{-9} \) m\(^2\)/s.

Fig. 5 depicts the average modulated signal (average release rate) and the average number of protons released by the cell according to (19) and (17), respectively, after excitation of the system with light intensity \( \mathcal{L}(t) = \mathcal{L}_\lambda u(t) \). Because of the active behavior of the pumps, for a given light intensity, the release rate approaches a constant steady state value after a transient time. Thereby, the total number of released molecules linearly increases in time. We also observe that increasing the light intensity from 50 to 500 mW/scm\(^2\) increases the average modulated signal to a lesser extend compared to increasing the light intensity from 5 to 50 mW/scm\(^2\). This behavior can be explained by considering that the round trip duration of the pumps depends on the return time to the resting state, \( X_{10} \), in addition to photon absorption time, \( X_{01}^p \). Since increasing the light intensity decreases the mean of \( X_{01}^p \) compared to that of \( X_{10} \), which is independent from the light intensity, for higher light intensities the term \( X_{01}^p \) has less impact on the average modulated signal than \( X_{10} \).

In Fig. 6, we show that the average concentration of the protons (i.e., the PH value) around the pumps remains in the optimal range for operating the pump during transmission of bit 1 which validates our assumption regarding the concentration outside the cell in Section II-B. The Bacteriorhodopsin pumping function is optimal at PH 6-7.5 [18]. The initial PH concentration is in the optimal range of operation of the pumps. Thereby, the assumption that the extracellular environment is validated.

For evaluation of the IPM performance, a simple spherical transparent receiver with radius \( r_{tx} = 5 \) \( \mu m \) is considered.

<table>
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<th>TABLE I</th>
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<td>THE TRANSITION RATES (RATE CONSTANTS (s(^{-1})) BETWEEN THE STATES OF THE BACTERIORHODOPSIN PHOTOCYCLE [18].</td>
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where the ions inside the receiver volume can be counted, without affecting the Brownian motion of the molecules [4]. The receiver is located at distance $d = 50 \mu m$ from the transmitter. For signal detection, the receiver takes $L$ samples of the number of observed ions during a time slot and the summation of the samples constitutes the receiver output, $y$, based on which a decision on the current symbol is made. Assume the diffusion channel has $J$ bits memory for time slot duration $T$. Given $J$ previous bits $B_j = b_j$, $j \in \{1, \cdots, J\}$, the Maximum-A-Posteriori (MAP) detector leads to a threshold based decision making for the current symbol, i.e., $B_0 = 0$, if $y \leq \text{Thr}$, and $B_0 = 1$, if $y > \text{Thr}$, (see [24]) where

$$\text{Thr} = \frac{\sum_{i=1}^L \mathbb{E}\{w(t_i)\} + \beta \mathbb{E}\{w(t_i)\}^{\ast} + \mathbb{E}\{w(t_i)\} \ast \mathbb{E}\{w(t_i)\}}{1 + \sum_{i=1}^L \beta_{j} \mathbb{E}\{w(t_i)\} \ast \mathbb{E}\{w(t_i)\}^{\ast} + \mathbb{E}\{w(t_i)\} \ast \mathbb{E}\{w(t_i)\}}. \quad (20)$$

Here, $p_{\text{obs}}(t)$ is the pdf of the observation time of an individual molecule at the receiver. As the decoder does not know the correct values of the previous bits, to compute the threshold, the previous decisions of the receiver are used which corresponds to a decision-feedback detector.

Based on this detector model, in Fig. 7 the error probability of proposed IPM based on off-keying modulation is compared with that of the instantaneous Poisson release of the signaling molecules proposed in [8] as a function of $T$. All results were obtained with the PBS. For $T_1 = 2$ s, more molecules are released from the cell than for $T_1 = 0.2$ s which leads to a better performance. As expected, the idealized instantaneous release of the molecules results in a more optimistic error probability, especially for larger values of $T_1$.

V. CONCLUSIONS

In this paper, a bio-synthetic modulator model for DMC based on ion pumps was proposed. Thereby, the release rate of the ions from the transmitter was controlled by modulating a light intensity signal. The pumping process of an individual light pump was modeled as a Markov process. Employing this model, the modulated signal of the IPM modulator was shown to be a non-homogeneous point process. A simple onoff keying modulation format based on the proposed IPM modulator was adopted for evaluation of the proposed modulator. Our simulation results reveal that the idealized assumption of instantaneous release of the molecules may lead to optimistic performance results compared to a biologically realizable transmitter.