Link Allocation for Multiuser Systems with Hybrid RF/FSO Backhaul: Delay-Limited and Delay-Tolerant Designs

Vahid Jamali, Student Member, IEEE, Diomidis S. Michalopoulos, Murat Uysal, Senior Member, IEEE, and Robert Schober, Fellow, IEEE

Abstract—In this paper, we consider a cascaded radio frequency (RF) and hybrid RF/free space optical (FSO) system where several mobile users transmit their data over an RF link to a decode-and-forward relay node (e.g., a small cell base station) and the relay forwards the information to a destination (e.g., a macro cell base station) over a hybrid RF/FSO backhaul link. The relay and the destination employ multiple antennas for transmission and reception over the RF links while each mobile user has a single antenna. The RF links are orthogonal to the FSO link but half-duplex with respect to each other, i.e., either the user-relay RF link or the relay-destination RF link is active. For this communication setup, we derive the optimal fixed and adaptive link allocation policies for sharing the transmission time between the RF links based on the statistical and instantaneous channel state information (CSI) of the RF and FSO links, respectively. Thereby, we consider the following two scenarios depending on the delay requirements: i) delay-limited transmission where the relay has to immediately forward the packets received from the users to the destination, and ii) delay-tolerant transmission where the relay is allowed to store the packets received from the users in its buffer and forward them to the destination when the quality of the relay-destination RF link is favorable. Our numerical results illustrate the effectiveness of the proposed communication architecture and link allocation policies, and their superiority compared to existing schemes which employ only one type of backhaul link.

Index Terms—Hybrid RF/FSO, backhaul link, adaptive/fixed link allocation, throughput, and delay.

I. INTRODUCTION

FREE space optical (FSO) communication has recently re-emerged as an attractive option for wireless backhauling due to its large usable bandwidth compared to traditional radio frequency (RF) backhauling [1]–[3]. Additionally, the narrow beams employed by FSO transceivers facilitate secure and interference-free communication. However, these beneficial properties of FSO systems come at the expense of some drawbacks and challenges. In particular, FSO communication relies on the availability of a line-of-sight (LOS) which introduces critical limitations for mobile nodes. Furthermore, FSO systems have an unpredictable connectivity due to atmospheric impairments such as atmospheric turbulence and visibility limiting conditions including snow, fog, and dust. Finally, even if LOS communication can be established, the pointing of the transmitter toward the photodetector may have to be adaptively adjusted to mitigate effects such as building sway [1], [2].

A possible strategy to mitigate the unpredictable connectivity of FSO links is so-called hybrid RF/FSO where an RF link is employed as a back-up for the FSO link [4]–[9]. This option is particularly attractive due to the fact that the impairments in the RF and FSO links are caused by different phenomena. Hence, a hybrid RF/FSO link is much more likely to maintain connectivity than a pure FSO link. One simple scheme to integrate RF and FSO systems is to transmit the same data over both links [4]. Based on this approach, hard switching [5] and soft switching [6] protocols were proposed where the more reliable link is selected for data detection. Furthermore, adaptive power control and adaptive combing of the RF and FSO signals was proposed in [7]. However, sending the same data over both the RF link and the FSO link does not efficiently exploit the available resources and is not an optimal transmission strategy. Hence, the authors in [9] modeled the hybrid RF/FSO system as two independent parallel channels and developed a joint encoding and decoding scheme. In addition to the aforementioned theoretical studies, experimental results obtained with hybrid RF/FSO testbeds were reported in [10], [11].

FSO transceivers are mainly attractive for nodes with fixed locations where a LOS link can be established. Motivated by this limitation of FSO for networks with mobile nodes, mixed RF/FSO systems have been proposed in the literature [12]–[17]. Here, the RF and FSO links are cascaded, i.e., the mobile nodes employ RF links to send their information to an intermediate fixed node, a relay node, and the intermediate node forwards the information to the final destination via an FSO backhaul link. This communication setup can model several practical applications including: i) Cellular communication where the mobile nodes send their data to a relay station and the relay station forwards the data to the base station; ii) Small cell networks where the mobile nodes in a building floor send their data to an access point and the access point forwards the information to the macro base station [2]; iii) The TV white space framework where the slave white space devices...
(WSDs) send their data to a master WSD and the master WSD forwards the data to the white space database [18]. The performance of mixed RF/FSO systems was investigated in [12]–[14] for a single-user amplify-and-forward (AF) relay network, and in [15], for a multi-user decode-and-forward (DF) relay network. Additionally, the effect of pointing errors in mixed RF/FSO systems was studied in [13] and [16] and the effect of imperfect channel state information (CSI) was investigated in [17].

In general, the end-to-end performance of dual-hop communication is limited by the weakest link. Hence, in mixed RF/FSO systems, atmospheric turbulence may lead to a significant degradation of the end-to-end performance, e.g., a high error floor or a limited end-to-end capacity. Motivated by this limitation, in this paper, we consider a cascaded (mixed) RF and hybrid RF/FSO system where an additional RF backhaul link is employed to support the FSO backhaul link. Thereby, we assume that the relay node employs decode-and-forward relaying for forwarding the users’ data to the destination and the back-up RF link for the relay-destination hop utilizes the same frequency band as the RF links for the user-relay communication. We consider a multi-user multiple-input multiple-output (MIMO) setup to fully exploit the available RF bandwidth. That is, relay and destination are equipped with multiple antennas for data transmission over the RF links. For simplicity and feasibility reasons, the relay is assumed to operate in the half-duplex mode with respect to the RF links, i.e., at a given time, the relay can either receive from the users or transmit to the destination over the RF links1. Moreover, we consider the following two scenarios depending on the delay requirements: i) delay-limited transmission where the relay has to immediately forward the packets received from the users to the destination, and ii) delay-tolerant transmission where the relay is allowed to store the packets received from the users in its buffer and forward them to the destination when the quality of the relay-destination RF link is favorable. For this system architecture and the aforementioned scenarios, we develop optimal fixed and adaptive link allocation policies which allocate transmission time to the RF links based on the statistical and instantaneous CSI of the RF and FSO links, respectively. The four proposed link allocation policies, namely delay-tolerant adaptive link allocation (DT-AL), delay-limited adaptive link allocation (DL-AL), delay-tolerant fixed link allocation (DT-FL), and delay-limited fixed link allocation (DL-FL), provide a trade-off between the achievable throughput, the delay, and the required signaling overhead. Thereby, the DT-AL policy provides an upper bound for the achievable throughput when the delay requirement is not stringent and the signaling overhead required for collecting instantaneous CSI can be afforded. In particular, the optimal DT-AL protocol switches adaptively between transmission and reception for the RF links in each transmission block which requires the relay to be equipped with a buffer [20]–[22] to temporarily store the data received from the users before forwarding it later over the FSO and/or RF backhaul links to the destination.

1Full-duplex RF relays have been reported in the literature [19]. However, they entail high hardware complexity for efficient self-interference suppression. Hence, in this paper, we focus on half-duplex RF relaying.

On the other hand, if the delay requirement is stringent and the signaling overhead required for collecting instantaneous CSI cannot be afforded, the proposed DL-FL policy has to be adopted. Thereby, the DL-FL protocol allocates a fixed fraction of the transmission blocks to the relay-destination RF link in order to maximize the end-to-end throughput under the given constraints. The throughputs of the DL-AL and DT-FL policies fall between those of the DT-AL and DL-FL policies. Our numerical results confirm the effectiveness of the proposed system architecture and link allocation policies, and their superiority compared to existing schemes which employ only one type of backhaul link.

In the following, we highlight the contributions of this paper compared to the recent paper [23] and our related conference papers [24], [25] which consider similar network architectures. In particular, the authors in [23] consider a three-node network comprised of a single-antenna source, a single-antenna relay, and a single-antenna destination where a direct RF link between the source and the destination is available. Unlike this paper, the focus of [23] is not on link allocation and the transmission time intervals allocated to the source-relay and relay-destination RF links are assumed to be predetermined. Instead, the authors of [23] develop a quantize-and-encode relaying scheme where the relay estimates and quantizes the log-likelihood ratio of each received bit in a symbol and transmits the corresponding information to the destination through a high speed FSO link or a hybrid RF/FSO link. In [24], we study the outage probability of a system similar to the one considered here. However, unlike in this paper, a heuristic transmission strategy with predefined fixed RF link assignment is considered in [24]. In [25], we consider the same system model as in this paper and derive the optimal DT-AL policy. However, alternative design options for different delay and/or CSI requirements, i.e., the DT-FL, DL-AL, and DL-FL policies, are not provided in [25].

The remainder of this paper is organized as follows. In Section II, the preliminaries and assumptions are presented. The proposed protocol, link capacities, and CSI requirements are discussed in Section III. In Section IV, the optimal link allocation policies are derived for different delay and CSI requirements. Numerical results are provided in Section V, and conclusions are drawn in Section VI.

Notations: We use the following notations throughout this paper: \( \mathbb{E}\{\cdot\} \) denotes expectation, \(| \cdot |\) represents the magnitude of a complex number and the determinant of a matrix, \( \mathcal{L} \) denotes the phase of a complex number, and \( \text{erf}(\cdot) \) is the Gauss-error function. Bold capital and small letters are used to denote matrices and vectors, respectively. \( A^H \) denotes the Hermitian transpose of \( A \), \( I_n \) is an \( n \times n \) identity matrix, \( \text{diag}(a_1, a_2, \ldots, a_n) \) is an \( n \times n \) diagonal matrix with main diagonal elements \( a_1, a_2, \ldots, a_n \), and \( [A]_{mn} \) denotes the element in the \( m \)-th row and \( n \)-th column of matrix \( A \). \( 1|\cdot| \in (0,1) \) is an indicator function which is equal to one if the argument is true and equal to zero if it is not true. Additionally, Rayleigh(\( \Phi \)), Rice(\( \Omega, \Psi \)), Unif(\( a,b \)), and GGamma(\( a, \beta \)) denote a Rayleigh random variable (RV) with parameter \( \Phi \), a Rician RV with parameters \( \Omega \) and \( \Psi \), a uniform RV uniformly distributed in the interval \([a,b]\), and a Gamma-Gamma RV with parameters
\[ \alpha \text{ and } \beta, \text{ respectively. Furthermore, } \mathbb{C}, \mathbb{R}, \text{ and } \mathbb{R}^+ \text{ are the sets of complex, real, and positive real numbers, respectively. For notational convenience, we use the definitions } C(x) \doteq \log_2(1 + x), \text{ } [x]_a \doteq 10 \log_{10} x, \text{ } [x]_a^b \doteq \min\{\max\{x,a\},b\}, \text{ and } [x]^+ \doteq \max\{0,x\}. \]

### II. Preliminaries and Assumptions

In this section, we describe the considered system model and introduce the model for the cascaded RF and hybrid RF/FSO communication links.

#### A. System Model

The considered system architecture is schematically shown in Fig. 1. In particular, \( K \) users \( U_k, \ k = 1, \ldots, K, \) wish to communicate with destination \( D \) via decode-and-forward relay node \( R \). There is no direct communication link between the users and the destination, i.e., the users can send their data to the destination only through the relay node. There are two types of communication links in our system model: \( i) \) the \( U-R \) and \( R-D \) RF links and \( ii) \) the \( R-D \) FSO link. All RF links use the same frequency band. We assume that each user has a single antenna while the relay and the destination are equipped with \( J \geq K \) and \( L \) antennas, respectively. Furthermore, we consider the practical half-duplex constraint for the relay node, i.e., the relay cannot simultaneously receive from the users through the \( U-R \) RF fronthaul/access link and transmit to the destination via the \( R-D \) RF backhaul link. On the other hand, since the FSO link does not interfere the RF links, the relay can always transmit data to the destination via the FSO backhaul link.

Let \( T_{UR}, T_{RD}^{RF}, \text{ and } T_{RD}^{FSO} \) denote the coherence times of the \( U-R \) RF link, the \( R-D \) RF link, and the \( R-D \) FSO link, respectively [26]. Since relay and destination are fixed nodes, the slow fluctuations in the \( R-D \) RF link are caused by the motion of objects in the environment, whereas the slow fluctuations in the \( R-D \) FSO link are due to atmospheric turbulence and scintillation [1], [26]. The time variance of the \( U-R \) RF link is introduced by the mobility of the users, and the corresponding coherence time is typically much smaller than those of the \( R-D \) RF and FSO links. Hence, in this paper, we assume that \( T_{UR} < T_{RD}^{RF}, T_{RD}^{FSO} \) holds and consider the coherence time of the \( U-R \) RF link as the time reference for link allocation. In particular, we assume the entire time of operation is divided into \( B \) blocks of length \( T_{UR} \). Moreover, each block includes \( N \) symbol intervals of the RF signals and each node transmits a codeword which spans one block or a fraction of one block. Hence, the symbol rate of the RF signals is given as \( R_{RF}^n = \frac{N}{T_{UR}} = \frac{W_{RF}}{f} \) where \( W_{RF} \) is the available bandwidth of the RF links. Furthermore, we assume that the bandwidth of the FSO link, \( W_{FSO}^n \), is \( M \) times larger than that of the RF links, i.e., \( W_{FSO}^n = M W_{RF}^n \). Hence, the duration of one symbol interval of the FSO signal is \( M \) times smaller than that of the RF signals, i.e., \( R_{FSO}^n = \frac{M N}{T_{UR}} = M R_{RF}^n \). Additionally, we assume that users always have enough information to send in all blocks and the number of blocks satisfies \( B \to \infty \). These assumptions are schematically illustrated in Fig. 2.

#### B. Communication Links

In the following, we describe the channel models for the RF and FSO links which are used throughout the paper.

**RF Links:** The \( U-R \) RF link constitutes an \( (K,J) \)-distributed MIMO system with \( K \) single-antenna transmitters and one \( J \)-antenna receiver. In contrast, the \( R-D \) RF backhaul link is a standard \((J,L)\)-point-to-point MIMO system. The received RF signals at the relay and the destination can be modelled as

\[
y_t^n[b] = H_t^n b \mathbf{x}_t^n[b] + \mathbf{z}_t^n[b], \quad t = 1, 2, \quad (1)
\]

where \( \mathbf{x}_t^n[b] \in \mathbb{C}^{K \times 1} \) and \( \mathbf{z}_t^n[b] \in \mathbb{C}^{J \times 1} \) denote the transmit symbol vectors of the users and the relay, respectively. \( \mathbf{y}_t^n[b] \in \mathbb{C}^{J \times 1} \) and \( \mathbf{y}_t^n[b] \in \mathbb{C}^{L \times 1} \) are the received symbol vectors at the relay and the destination, respectively, and \( \mathbf{z}_t^n[b] \in \mathbb{C}^{L \times 1} \) and \( \mathbf{z}_t^n[b] \in \mathbb{C}^{L \times 1} \) denote the noise vectors at the relay and the destination, respectively, in the \( n \)-th symbol interval of the \( b \)-th block. We assume that \( \mathbf{z}_t^n[b] \) and \( \mathbf{z}_t^n[b] \) are zero-mean complex additive white Gaussian noise (AWGN) vectors with covariance matrices \( \sigma^2_{\mathbf{x}} \mathbf{I}_J \) and \( \sigma^2_{\mathbf{z}} \mathbf{I}_L \), respectively, and are mutually independent and independent from the transmitted codewords. The variance of the noise at the RF receivers is given by \( \sigma^2_{\mathbf{z}} = W_{RF} \sigma^2_{\mathbf{x}} \) where \( \sigma^2_{\mathbf{x},0} \) and \( \sigma^2_{\mathbf{x},f} \) denote the noise power spectral density (in dB/Hz) and the noise figure (in dB) of the receiver, respectively. Furthermore, \( H_t^n[b] \in \mathbb{C}^{J \times K} \) and \( H_t^n[b] \in \mathbb{C}^{L \times J} \) denote the channel coefficient matrices of the \( U-R \) and \( R-D \) RF links, respectively. Moreover, we assume that all elements of \( H_t^n[b] \)
and $H_2[b]$ are mutually independent, ergodic, and stationary random processes with continuous probability density functions. Throughout the paper, we use the following statistical models for the $U$-$R$ and $R$-$D$ RF links.

**U-$R$ RF Link**: Due to the mobility of the users, we assume that there is no LOS link available between the users and the relay. Therefore, we assume Rayleigh fading for the $U$-$R$ RF link [27]. Let $h_{1,k}^j$ denote the element in the $j$-th row and $k$-th column of matrix $H_1[b]$. $h_{1,k}^j[b]$ can be written as $h_{1,k}^j[b] = \sqrt{h_{1,k}^j \eta_{1,k}^j[b]}$ [23], where $h_{1,k}^j$ and $\eta_{1,k}^j[b]$ are respectively the average power gain and the fading coefficient of the RF link between the $k$-th user and the $j$-th antenna at the relay node and are given by [27, page 20]

$$h_{1,k}^j = \left(\frac{d_{RF}^j}{4\pi d_{RF}^j}ight)^2 \times \left[\frac{d_{RF}^j}{d_{RF}^j}ight] \nu$$

$$|h_{1,k}^j[b]| \sim \text{Rayleigh}(\Phi), \quad \angle h_{1,k}^j[b] \sim \text{Unif}(-\pi, \pi),$$

where $\lambda_{RF}$ is the wavelength of the RF signal, $G_{U}^{R}$ and $G_{R}^{R}$ are the RF transmit antenna gain of the users and the receive antenna gain of the relay, respectively, $d_{RF}^j$ is a reference distance for the antenna far-field, $d_{TU,R}$ is the distance between the $k$-th user and the relay, and $\nu$ is the RF path-loss exponent. Parameter $\Phi$ in the Rayleigh distribution is the power of the fading term $h_{1,k}^j[b]$, i.e., $\Phi = \mathbb{E}[|h_{1,k}^j[b]|^2] = 1$.

**R-$D$ RF Link**: In a hybrid RF/FSO backhaul link, a LOS has to be available for the applicability of the FSO system. Therefore, we assume Rician fading for the $R$-$D$ RF link which models the effects of both scattered and LOS paths [27]. Assume that $h_{2}^j[b]$ denotes the element in the $j$-th row and $j$-th column of matrix $H_2[b]$ and can be written as $h_{2}^j[b] = \sqrt{h_{2}^j \eta_{2}^j[b]}$ [23], where $h_{2}^j$ and $\eta_{2}^j[b]$ are respectively the average power gain and the fading coefficient of the RF link between the $j$-th antenna at the relay and the $j$-th antenna at the destination, respectively, and are given by [27, page 23]

$$h_{2}^j = \left(\frac{\lambda_{RF} d_{RD}^{\text{ref}}}{4\pi d_{RD}^{\text{ref}}}ight)^2 \times \left[\frac{d_{RF}^j}{d_{RD}} \frac{d_{RD}^{\text{ref}}}{d_{RD}^{\text{ref}}}ight] \nu$$

$$|h_{2}^j[b]| \sim \text{Rice}(\Omega, \Psi), \quad \angle h_{2}^j[b] \sim \text{Unif}(-\pi, \pi)$$

Here, $G_{R}^{D}$ is the RF transmit antenna gain of the relay and $d_{RD}$ is the distance between relay and destination. In the Rice distribution, parameter $\Omega$ is the ratio between the power in the direct path and the power in the scattered paths, and parameter $\Psi$ is the total power in both paths, i.e., $\Psi = 1$.

Finally, the transmitted RF signals have to meet the following per-node average power constraints in each symbol interval

$$\mathbb{E}[x_k^j[b]|x_k^j[b]H] \leq \text{diag}(p_{RF}^{U_1}, p_{RF}^{U_1}, \ldots, p_{RF}^{U_k})$$

(4a)

$$\mathbb{E}[x_k^j[b]H x_k^j[b]] \leq p_{RF}^{R}$$

(4b)

where $p_{RF}^{U_k}$ and $p_{RF}^{R}$ are the maximum RF transmit powers of user $k$ and the relay, respectively.

**FSO Link**: The relay node is equipped with an aperture transmitter pointing in the direction of a photodetector at the destination. We assume an intensity modulation direct detection (IM/DD) FSO system with on-off keying (OOK) modulation and soft decoding, i.e., the photodetector directly detects the intensity of the received photocurrent by integrating over each symbol interval. In particular, after removing the ambient background light intensity, the detected signal intensity at the destination is modelled as [29], [30]

$$y^m[b] = g[b]x^m[b] + z^m[b],$$

(5)

where $x^m[b] \in [0, P_{FSO}^{RF}]$, $y^m[b] \in \mathbb{R}$, and $z^m[b] \in \mathbb{R}$ are the power of the OOK modulated symbol at the relay, the detected signal at the destination, and the shot noise at the destination caused by ambient light for the $m$-th symbol of the $b$-th block, respectively. Here, $P_{FSO}^{RF}$ denotes the maximum allowable transmit power of the relay in the FSO link which is mainly determined by eye safety regulations [1]. Noise $z^m[b]$ is modelled as zero-mean real AWGN with variance $\sigma^2$ and is independent of the transmitted signal [1, Sections II.D and III.C]. Moreover, $g[b] \in \mathbb{R}^+$ is modelled as $g[b] = g_a g_f[b]$ [8], [23], where $g_a$ and $g_f[b]$ are the average gain and the fading gain of the FSO link, respectively, and are given by [6], [8]

$$g_a = \rho \left[\text{erf}\left(\frac{\sqrt{\pi} r}{2\phi d + \pi \kappa d}ight)\right]^{2} \times 10^{-2\kappa \pi d/10}$$

$$g_f[b] \sim \text{GGamma}(\alpha, \beta)$$

where $\rho$ is the responsivity of the photodetector, $r$ is the aperture radius, $\phi$ is the divergence angle of the beam, and $\kappa$ is a weather-dependent attenuation coefficient. We adopt the Gamma-Gamma fading model, i.e., we assume that the large-scale and small-scale irradiance fluctuations are both governed by Gamma distributions [31]. Therefore, parameters $\alpha$ and $\beta$ in the Gamma-Gamma distribution are in fact the inverse of the variances of the large-scale and small-scale turbulent fading terms. Assuming spherical wave propagation, parameters $\alpha$ and $\beta$ in the Gamma-Gamma distribution are related to physical parameters as follows [8], [31]

$$\alpha = \exp\left\{0.49 \theta^2 \left[1 + 0.18 \epsilon^2 + 0.56 \theta^{12/5} \right]^{-1}\right\}$$

(7a)

$$\beta = \exp\left\{0.51 \theta^2 \left[1 + 0.69 \theta^{12/5} \right]^{-5/6}\right\} \left(1 + 0.9 \epsilon^2 + 0.62 \theta^{12/5} \right)^{1/6}\right\}^{-1},$$

(7b)

where $\theta^2 = 0.5C_5^{2,-7/6}(d_{RD})^{11/6}$, $\epsilon^2 = \sigma^2/d_{RD}$, and $\varsigma = 2\pi/\lambda_{FSO}$. Here, $\lambda_{FSO}$ is the wavelength and $C_5^{\theta}$ is the weather-dependent index of refraction structure parameter.

**III. PROPOSED PROTOCOL, LINK CAPACITIES, AND CSI REQUIREMENTS**

In this section, we first introduce the considered protocol, review the link capacities, and explain the adopted coding schemes. Subsequently, we specify our assumptions regarding the required CSI knowledge.
A. The Proposed Protocol

Recall that due to the half-duplex constraint, the relay cannot use the U-R and R-D RF links simultaneously. In fact, if the quality of the FSO backhaul link is sufficiently good, all data received at the relay from the users can be forwarded over the FSO link to the destination, and there is no need to activate the R-D RF backhaul link at all. However, due to turbulence in the FSO link, the relay may not always be able to forward the users’ information to the destination by employing only the FSO link. Hence, the R-D RF link is needed as a back-up. On the other hand, using the R-D RF backhaul link comes at the expense of reducing the time of transmission of the users to the relay over the U-R RF link due to the half-duplex constraint.

In light of the above discussion, the main idea of the proposed protocol is to optimally divide the transmission time between the U-R RF fronthaul and the R-D RF backhaul links. Next, we introduce a general notation which is then used for the DT-AL, DL-AL, DT-FL, and DL-FL policies. In particular, we introduce time sharing variable $q[b] \in [0, 1]$ where $q[b]$ denotes the fraction of block $b$ in which the relay receives, i.e., the U-R RF link is active. In the remaining $1 - q[b]$ fraction of block $b$, the relay transmits over the R-D RF link. We note that the optimal value of $q[b]$ for the DT-AL and DL-AL policies depends on the instantaneous and/or the statistical CSI of the involved links, see Table I, and hence it changes from one transmission block to the next. On the other hand, the optimal value of $q[b]$ for the DT-FL and DL-FL policies depends only on the statistical CSI of the involved links, see Table I, and is independent of the instantaneous CSI in each transmission block. In particular, $q[b] = q$ holds for $\forall b$, in the DL-FL policy, i.e., the first $q$ fraction of each transmission block is allocated to the U-R RF link and the remaining $1 - q$ fraction of each transmission block is allocated to the R-D RF link. In contrast for the DT-FL policy, the U-R RF link or the R-D RF link are selected for transmission for several consecutive blocks to average out the fading. To this end, for the DT-FL policy, the total $B$ blocks are divided into $G$ groups where each group consists of $B' = B/G$ blocks. Moreover, without loss of generality, we assume that $B, B', G \to \infty$. Thereby, the U-R RF link is selected for the first $qB'$ blocks of each group and the R-D RF link is employed for the remaining $(1 - q)B'$ blocks. Hence, for the DT-FL policy, we assume $q[b] = 1$ holds for the first $qB'$ blocks and $q[b] = 0$ holds for the remaining $(1 - q)B'$ blocks of each group.

Let $Q[b]$ denote the number of information bits available in the buffer after receiving the information from the users in the $b$-th block. Note that tracking the amount of information bits in the buffer of the relay is crucial for the delay-tolerant policies, i.e., DT-AL and DT-FL, whereas for the delay-limited policies, i.e., DL-AL and DL-FL, the relay forwards the information in its queue immediately after it receives them. The optimization of the proposed policies, i.e., the optimization of $q[b]$, $\forall b$, will be discussed in detail in Section IV. However, first the code schemes, the transmission rates, and the dynamics of the queues at the buffers are presented in the next subsection.

B. Link Capacities and Queue Dynamics

1. Link Capacities: The users employ Gaussian codebooks, i.e., the $k$-th element of vector $x_{\ell}^k[b]$ is generated independently according to a zero-mean rotationally invariant complex Gaussian distribution with variance $\frac{P_{\text{RF}}}{N_{\text{RF}}}$. At the beginning of each fading block, each user $k$ encodes $q[b]N_{\text{RF}}$ bits of information into a codeword with length $q[b]N$ symbols taken from a Gaussian codebook with a fixed rate $R_{\text{RF}}$ bits/symbol. The users transmit their codewords and the relay receives $y_{\ell}^k[b]$ according to (1). The relay can employ several multi-user detection schemes proposed in the literature [15], e.g., linear zero-forcing (ZF) and minimum mean square error (MMSE) detection, nonlinear detection schemes incorporating successive interference cancellation (SIC), and optimal maximum likelihood (ML) detection. Depending on the type of detector used, the codeword of each user experiences a certain signal-to-noise ratio (SNR) denoted by $\gamma_{\text{RF}}^k[b]$. For instance, for ZF detection, $\gamma_{\text{RF}}^k[b]$ is given by

$$\gamma_{\text{RF}}^k[b] = \frac{\frac{P_{\text{RF}}}{N_{\text{RF}}}}{\sigma_{b,k}^2} \left( \frac{1}{\left( (H_{\ell}[b])^\dagger H_{\ell}[b] \right)^{-1}} \right)_{kk}.$$  (8)

The codeword of user $k$ can be decoded reliably only if $\gamma_{\text{RF}}^k[b] \geq 2R_{\text{RF}} - 1$; otherwise the relay cannot decode the codeword and has to drop it for delay-limited applications or ask user $k$ to retransmit this information in the following blocks for delay-tolerant applications. The effective rate of the U-R RF link is defined as the normalized sum of the information bits in bits/symbol that can be decoded successfully at the relay and is given by

$$C_{\text{RF}}^k[b] = \sum_{b=1}^{B} \left\lfloor \gamma_{\text{RF}}^k[b] \geq 2R_{\text{RF}} - 1 \right\rfloor R_{\text{RF}}.$$  (9)

2. RF Link: This backhaul link is a standard point-to-point MIMO channel with an average power constraint across all antennas. We employ Gaussian codebooks, i.e., vector $x_{\ell}^k[b]$ is a multivariate zero-mean rotationally invariant complex Gaussian vector, and waterfilling power allocation across the transmit antennas [32]. In order to ensure reliable detection at the destination, the relay cannot transmit more information bits over the R-D RF link than this link can support, i.e.,

$$C_{\text{RF}}^k[b] = \min_{J=L} \left\lfloor \log_2 \left( \frac{\mu[b]}{\sqrt{\frac{\sigma_{\text{RF}}^2}{2}}} \right) \right\rfloor.$$  (10)

Here, $\chi_{\ell}^k[b]$ is the $j$-th singular value of $H_{\ell}[b]$ [32]; $\mu[b]$ is the water level which is chosen to satisfy the power constraint in (4b) as the solution of the following equation

$$\sum_{j=1}^{\min(J,L)} \left( \frac{\mu[b]}{\sigma_{\text{RF}}^2} \chi_{\ell}^k[b] \right) = P_{\text{RF}}.$$  (11)

Furthermore, the relay cannot transmit more information bits
over the RF link than what it has stored in its buffer and has not yet been transmitted over the FSO link, i.e., $Q[b]$. Hence, the relay extracts $\min \{Q[b], (1 - q[b])NC^2_{\text{RF}}[b]\}$ bits of information from its buffer and transmits them over the $J$ antennas of the RF link in the $1 - q[b]$ remaining fraction of the $b$-th block.

**FSO Link:** We employ OOK modulation and soft detection for the FSO link. From an information theoretical point of view, the FSO link can be modelled as a binary input-continuous output AWGN channel where the capacity is achieved with a uniform distribution of the binary inputs [33], [34]. For reliable detection at the destination, the relay cannot transmit more than $MNC^{FSO}[b]$ information bits over the FSO channel in the $b$-th block, where $C^{FSO}[b]$ is the capacity of the Gaussian channel with OOK input and is given by [33], [34]

$$C^{FSO}[b] = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp \left\{ -w^2 \right\} \log_2 \left\{ 1 + \exp \left\{ -\frac{p[b]}{\sigma^2} \right\} \right\} \text{d}w,$$

where $p[b] = p_g[b]^{FSO}$. Additionally, the relay cannot transmit more information bits over the FSO link than what it has stored in its buffer at the beginning of the $b$-th block, i.e., $[Q[b-1] - (1 - q[b-1])NC^{RF}_2[b-1]^+]$. Hence, the relay extracts $\min \{[Q[b-1] - (1 - q[b-1])NC^{RF}_2[b-1]^+], MNC^{FSO}[b]\}$ bits of information from its buffer, encodes them into a codeword, and sends the codeword over the RF/FSO link to the destination in the $b$-th transmission block.

**Dynamics of the Queue:** After the relay has received the information from the users in the $b$-th block, the amount of information bits in the buffer is updated as

$$Q[b] = q[b]NC^{RF}_2[b] + \begin{cases} [Q[b-1] - (1 - q[b-1])NC^{RF}_2[b-1] \quad &\text{DT-AL and DL-FL} \\ -MNC^{FSO}[b] \quad &\text{DL-FL} \end{cases},$$

where $b \in \{2,3,\ldots,B\}$ and $Q[1] = q[1]NC^{RF}_2[1]$. We note that for delay-limited applications, if $Q[b-1] - (1 - q[b-1])NC^{RF}_2[b-1] > MNC^{FSO}[b]$ holds, the relay has to drop the information bits remaining in its queue which it is not able to forward to the destination.

**C. CSI Knowledge**

Throughout this paper, we assume that the relay is responsible for determining the optimal transmission strategy and for conveying the strategy to the other nodes. Moreover, the CSI knowledge required at the relay depends on the adopted link allocation policy, see Table I. Furthermore, since for large $K$, an excessive amount of CSI feedback would be required for adaptive rate transmission at the user nodes, for all proposed policies, we assume that the users transmit with a priori fixed transmission rates and hence, no CSI knowledge is required at the users. Furthermore, for all proposed policies, the destination knows the CSI of the RF/FSO links as required for reliable coherent decoding. For the adaptive link allocation policies, i.e., DT-AL and DL-AL, we assume that the channel states change slowly enough such that the signaling overhead caused by channel estimation and feedback is negligible compared to the amount of information transmitted in one block. Additionally, as stated in Section II.A, we assume that $T_{ULR} < T_{RF}^{FSO}$ holds which is a practical assumption since the users are mobile nodes whereas both the relay and the destination are fixed nodes. We note that this assumption is required for the DL-AL policy, cf. Theorem 2, whereas for the other policies, it is sufficient that $T_{ULR} < T_{RD}^{FSO}$ holds, cf. Theorem 1 and Propositions 1 and 2.

**IV. LINK ALLOCATION FOR THE CASCADED RF AND HYBRID RF/FSO CHANNEL**

In this section, we derive the optimal link allocation policies for the proposed transmission protocol for different delay requirements and different CSI requirements.

**A. Adaptive Link Allocation**

In this section, our goal is to specify the optimal adaptive link allocation, i.e., the optimal $q[b], \forall b$, for delay-tolerant and delay-limited transmission such that the average number of information bits received at the destination, denoted by $\tau$, is maximized. Note that delay-tolerant transmission can offer a higher average data rate compared to delay-limited transmission at the expense of an increased end-to-end delay. Both delay-limited and delay-tolerant transmission have practical relevance, e.g., file downloading is an application suited for delay-tolerant transmission and real-time voice chatting is an application requiring delay-limited transmission.

Recall that there is no direct link, hence, the amount of information received from the users at the destination is identical to that received from the relay at the destination. Therefore, the optimization problem for average throughput maximization is formulated as follows

$$\text{maximize } \tau = \lim_{B \to \infty} \frac{1}{B} \sum_{b=1}^{B} \left\{ \min \{Q[b], (1 - q[b])NC^{RF}_2[b]\} \right\} \text{d}Q[b], \quad \forall b$$

$$+ \min \{[Q[b-1] - (1 - q[b-1])NC^{RF}_2[b-1]^+]\}, MNC^{FSO}[b]) \right\}.$$

The expressions in the first and second min functions in the above equation are the numbers of information bits received from the relay at the destination over the RF and FSO channels in the $b$-th transmission block, respectively. In the following, we provide the optimal link allocation policies for the considered delay requirements as solutions to the above optimization problem.
Algorithm 1 Gradient algorithm for $\omega^*$

| initialize $i = 0$, $\omega[0] \in (0,1)$, and a desired small $\varepsilon > 0$
| repeat
| 1. Compute $c_{1,RF}^{RF}[i]$ and $c_{2,RF}^{RF}[i]$ from (18) for $\omega[i]$
| 2. Update $\omega[i+1]$ based on (17)
| 3. Set $i = i + 1$
| until $|\omega[i+1] - \omega[i]| < \varepsilon$

1) Delay-Tolerant Transmission: Finding the optimal delay-tolerant link allocation policy corresponding to problem formulation in (14) is challenging for the considered channel model due to the recursive dynamics of the queue, cf. (13). Nevertheless, we can obtain an upper bound on the achievable throughput of the proposed protocol by neglecting the effect of the queues on the transmission rate in (14).

**Upper Bound $\tau^{upp}$:** The achievable average sum throughput of the proposed protocol for the considered cascaded RF and hybrid RF/FSO system is upper bounded by $\tau^{upp}$ obtained from the following optimization problem

$$
\max \quad q(b) \epsilon [0,1], \forall b
$$

subject to

$$
C1: \tau^{upp} \leq \lim_{B \to \infty} \frac{1}{B} \sum_{b=1}^{B} q(b) NC_{1,RF}^{RF}[b]
$$

$$
C2: \tau^{upp} \leq \lim_{B \to \infty} \frac{1}{B} \sum_{b=1}^{B} [(1 - q(b)) NC_{2,RF}^{RF}[b] + NM C_{FSO}^{FSO}[b]].
$$

The above upper bound follows from the max-flow min-cut theorem [35, Section 15], where for a given sequence of $q[b] \epsilon [0,1], \forall b$, the information rate from the users to the destination is upper bounded by the minimum of the maximum information rates that can be transmitted over the user-relay and relay-destination hops. In the following theorem, we provide the optimal solution to the above optimization problem. For notational simplicity, let $C_{1,RF}^{RF}(H_1)$, $C_{2,RF}^{RF}(H_2)$, and $C_{FSO}^{FSO}(g)$ denote the capacity functions given in (9), (10), and (12) in terms of the fading states $H_1$, $H_2$, and $g$, respectively. Moreover, let $f_{H_1}(H_1)$, $f_{H_2}(H_2)$, and $f_g(g)$ denote the probability density functions of $H_1$, $H_2$, and $g$, respectively.

**Theorem 1:** The optimal DT-AL policy for the considered cascaded RF and hybrid RF/FSO system depends only on the values of channel matrices $H_1$ and $H_2$ in each fading block and is denoted by $q^*(H_1,H_2) \epsilon [0,1]$. Moreover, this optimal link allocation policy reduces to the adaptive activation of either the U-R link or the R-D RF link according to

$$
q^*(H_1,H_2) = \begin{cases} 
0, & \text{if } \omega C_{1,RF}^{RF}(H_1) < [1 - \omega] C_{2,RF}^{RF}(H_2) \\
1, & \text{otherwise}
\end{cases}
$$

where $\omega \epsilon (0,1]$ is a constant which does not depend on the instantaneous realization of the fading but depends on the distributions of the involved fading processes. The optimal value of $\omega$ can be obtained offline before transmission starts using Algorithm 1 with the following update equation

$$
\omega[i+1] = \left[ \omega[i] - \delta[i] \right] \left[ c_{1,RF}^{RF}[i] - C_{2,RF}^{RF}[i] - MC_{FSO}^{FSO} \right]_{\geq 0}^{1},
$$

where $i$ is the iteration index and $\delta[i]$ is an appropriately chosen step size parameter. The average capacity rates $c_{1,RF}^{RF}[i]$, $C_{2,RF}^{RF}[i]$, and $C_{FSO}^{FSO}$ are given by

$$
\begin{align*}
C_{1,RF}^{RF}[i] &= \mathbb{E}[q^*(H_1,H_2)c_{1,RF}^{RF}(H_1)] \\
&= \int_{H_1 \in C \times K} \int_{H_2 \in C \times L} q^*(H_1,H_2)c_{1,RF}^{RF}(H_1) \\
&\quad \times f_{H_1}(H_1)f_{H_2}(H_2)dH_1dH_2
\end{align*}
$$

$$
\begin{align*}
C_{2,RF}^{RF}[i] &= \mathbb{E}\left\{ [1 - q^*(H_1,H_2)]c_{2,RF}^{RF}(H_2) \right\} \\
&= \int_{H_1 \in C \times K} \int_{H_2 \in C \times L} [1 - q^*(H_1,H_2)]c_{2,RF}^{RF}(H_2) \\
&\quad \times f_{H_1}(H_1)f_{H_2}(H_2)dH_1dH_2
\end{align*}
$$

$$
C_{FSO}^{FSO} = \mathbb{E}\{c_{FSO}(g)\} = \int_{g \in \mathbb{R}^+} c_{FSO}(g)f_g(g)dg,
$$

where for a given $\omega = \omega[i]$ in the $i$-th iteration, $q^*(H_1,H_2)$ has to be obtained from (16) for a given set of fading values. Employing the optimal $\omega^*$ obtained from Algorithm 1 and the optimal $q^*(H_1,H_2)$ from (16), and substituting them into (18), the upper bound $\tau^{upp}$ is obtained as

$$
\tau^{upp} = N \min \left\{ c_{1,RF}^{RF}, c_{2,RF}^{RF} + MC_{FSO}^{FSO} \right\}.
$$

**Proof:** Please refer to Appendix A.

We note that the optimal policy in Theorem 1 is a function of $i) c_{1,RF}^{RF}(H_1)$ and $c_{2,RF}^{RF}(H_2)$ which require instantaneous CSI knowledge of $H_1$ and $H_2$, and $ii) \omega$ which depends on average capacity rates $c_{1,RF}^{RF}[i]$, $c_{2,RF}^{RF}[i]$, and $C_{FSO}^{FSO}$ given in (18) whose evaluation in turn requires statistical knowledge, i.e., knowledge of $f_{H_1}(H_1)$, $f_{H_2}(H_2)$, and $f_g(g)$. In the following, we formally state the achievability of the upper bound $\tau^{upp}$ as $B \to \infty$.

**Lemma 1:** The upper bound $\tau^{upp}$ is achievable if the optimal link allocation policy in Theorem 1 is employed. More precisely, as $B \to \infty$, we obtain $\tau = \tau^{upp}$.

**Proof:** Please refer to Appendix B.

We note that Lemma 1 reveals that $\tau^{upp}$ is asymptotically achievable by using the link allocation policy in Theorem 1, i.e., $\tau \to \tau^{upp}$ as $B \to \infty$. Thereby, it is possible that for some transmission blocks, the relay does not have enough data in its queue to send to the destination over the FSO and R-D RF links, but as $B \to \infty$, the effect of these events on the average throughput in (14) becomes negligible, i.e., $\tau \to \tau^{upp}$.

**Corollary 1:** The optimal value of Lagrange multiplier $\omega$ can be obtained analytically from one of the following mutually exclusive cases:

**Case 1:** If $\mathbb{E}\{c_{1,RF}^{RF}(H_1)\} \leq M\mathbb{E}\{c_{FSO}^{FSO}(g)\}$, we hold $\omega^* = 1$. Hence, we have $q^*(H_1,H_2) = 1, \forall H_1,H_2$.

**Case 2:** If $\mathbb{E}\{c_{1,RF}^{RF}(H_1)\} > M\mathbb{E}\{c_{FSO}^{FSO}(g)\}$, the optimal value of $\omega$ is obtained analytically from the following equation

$$
\int_{H_1 \in C \times K} \int_{H_2 \in C \times L} q^*(H_1,H_2)c_{1,RF}^{RF}(H_1) \\
\times f_{H_1}(H_1)f_{H_2}(H_2)dH_1dH_2
$$

4The integral equations in (18) can be solved numerically using mathematical software packages such as Mathematica. Alternatively, the equivalent expectations can be computed using Monte Carlo simulation, i.e., by averaging the arguments of the expectation over a large number of random realizations of $H_1$, $H_2$, and $g$. 


where $q^*(\mathbf{H}_1, \mathbf{H}_2)$ is substituted from (16).

Proof: Please refer to Appendix C.

Corollary 1 provides an important intuition regarding the impact of the statistics of the RF and FSO links. In particular, if the FSO link is statistically strong (cf. Case 1), the relay can forward all the information received from the users to the destination over the FSO backhaul link and the RF backhaul link remains inactive for all transmission blocks, i.e., we have $\omega^* = 1$. However, if the statistical quality of the FSO link is not sufficiently strong, e.g. due to adverse atmospheric conditions (cf. Case 2), the RF backhaul link becomes active in fading blocks which satisfy $\omega C_{RF}^1(\mathbf{H}_1) < [1 - \omega] C_{RF}^2(\mathbf{H}_2)$. Thereby, the optimal value of $\omega$ is obtained such that the sum of the average information rates sent from the relay to the destination over both the RF and the FSO links is identical to the average information rate received at the relay from the users.

2) Delay-Limited Transmission: For delay-limited transmission, the information received from the users at the relay node has to be immediately forwarded to the destination over the RF backhaul link in the current block and/or the FSO link in the subsequent block. Hence, $Q(b)$ in (13) is not recursively related to the previous buffer states and optimization problem (14) can be separated into independent optimization problems which are solved in each block, i.e.

$$
\text{maximize } \min \left\{ (1 - q(b)) N C_{RF}^2[b] + M N C_{FSO}[b+1], q(b) N C_{RF}^1[b] \right\}.
$$

The solution to the above problem is given in the following theorem.

**Theorem 2:** The optimal DL-AL policy for the considered cascaded RF and hybrid RF/FSO system is given by

$$
q^*[b] = \begin{cases} 
\frac{MC_{FSO}[b+1] + C_{RF}^2[b]}{C_{RF}^1[b] + C_{RF}^2[b]}, & \text{if } C_{RF}^1[b] > MC_{FSO}[b+1] \\
1, & \text{otherwise} 
\end{cases}
$$

(22)

The achievable throughput of the proposed protocol with the above optimal link allocation policy is given by

$$
\tau^* = N E\{q^*(\mathbf{H}_1, \mathbf{H}_2, g) C_{RF}^1(\mathbf{H}_1)\}
= N \int_{\mathbf{H}_1 \in \mathcal{C}_1^{J \times K}} \int_{\mathbf{H}_2 \in \mathcal{C}_1^{J \times J}} \int_{g \in \mathbb{R}^+} q^*(\mathbf{H}_1, \mathbf{H}_2, g) C_{RF}^1(\mathbf{H}_1) \times f_{\mathbf{H}_1}(\mathbf{H}_1) f_{\mathbf{H}_2}(\mathbf{H}_2) f_g(g) d\mathbf{H}_1 d\mathbf{H}_2 d g,
$$

(23)

where $q^*(\mathbf{H}_1, \mathbf{H}_2, g)$ is the optimal link allocation policy in (22) as a function of the fading states $\mathbf{H}_1, \mathbf{H}_2$, and $g$.

Proof: The first and the second term in the min function in (21) are monotonically decreasing and increasing functions in $q[b]$, respectively. If $C_{RF}^1[b] \leq MC_{FSO}[b+1]$ holds, the two terms in the min function in (21) cannot be made equal for any $q[b] \in (0, 1)$. Therefore, the solution must be at the boundary points, i.e., $q[b] = 0, 1$. Among these two points, $q[b] = 0$ leads to $\tau = 0$ and hence, $q^*[b] = 1$ is the optimal solution. On the other hand, if $C_{RF}^1[b] > MC_{FSO}[b+1]$ holds, the two terms in the min function in (21) coincide exactly at one point, i.e., $q^*[b] = \frac{MC_{FSO}[b+1] + C_{RF}^2[b]}{C_{RF}^1[b] + C_{RF}^2[b]}$. Furthermore, the link allocation policy in (22) guarantees that all data received from the users at the relay node will be forwarded to the destination. Hence, the average throughput is identical to the average data rate between the users and the relay, i.e., (23) holds. This completes the proof.

The optimal link allocation policy in Theorem 2 reveals that if the instantaneous quality of the FSO link is good enough such that all information received from the users at the relay node can be forwarded to the destination over the FSO link, i.e., if $C_{RF}^1[b] < MC_{FSO}[b+1]$ holds, the RF backhaul link is not activated. Otherwise, the transmission time has to be shared between the U-R and the RF links such that the rate of the U-R RF link becomes equal to the sum of the rates of the RF link and the FSO link.

**Remark 1:** For computation of $C_{FSO}[b+1]$, the optimal DT-AL policy in Theorem 2 requires non-causal CSI of the FSO link at the central node at the beginning of block $b$. Such non-causal information is not available in general, however, as stated in Section III.B, in the considered system model, the coherence times of the RF and FSO links are much larger than the duration of one block, i.e., $T_{UR} \ll T_{RF}$ where $T_{UR}$ is assumed to be the duration of one block. This is a practical assumption since the users are mobile nodes whereas the relay and the destination are fixed nodes. Hence, the CSI of the FSO link can be assumed to be valid for many transmission blocks. Thus, for this scenario, we can assume that the central node knows $C_{FSO}[b+1]$ at the beginning of block $b$ since $C_{FSO}[b+1] = C_{FSO}[b]$ during the coherence time of the FSO link.

**Remark 2:** The following interesting observations can be made by comparing the optimal DT-AL policy in Theorem 1 and the optimal DL-AL policy in Theorem 2: i) The optimal DL-AL policy depends on the instantaneous fading gains of both the RF and the FSO links whereas the optimal DT-AL policy depends on the instantaneous fading gains of the RF links only. ii) The optimal DL-AL policy does not depend on the statistics of the involved links whereas the optimal DT-AL policy depends on the statistics of both the RF and the FSO links through Lagrange multiplier $\omega$. iii) The optimal DL-AL policy allocates a fraction of the transmission time to the RF backhaul link in each block whereas the optimal DT-AL policy reduces to scheduling of either the U-R RF link or the RF link.

B. Fixed Link Allocation

The adaptive link allocation protocols proposed in the previous subsection are only applicable if instantaneous CSI is available at the relay node. This is a reasonable assumption if the coherence time of the U-R channels is sufficiently large such that the signaling overhead for CSI acquisition, determining the optimal link allocation, and feeding back the solution to the nodes is negligible compared to the total time.
left for data transmission. In this subsection, we derive fixed link allocation policies for the scenario where the overhead of adaptive link allocation cannot be accommodated. Thereby, the fixed link allocation policy depends on the statistics of the involved links and does not change as a function of the instantaneous CSI. We consider again delay-tolerant and delay-limited transmission, i.e., DT-FL and DL-FL, and find the corresponding optimal link allocation policies, i.e., \(q^*\).

1) Delay-Tolerant Transmission: Assuming \(B, B', G \to \infty\) such that \(B' = \frac{B}{G}\) holds, the optimization problem for the optimal DT-FL policy is formulated as follows

\[
\text{maximize } \tau = N \min_{q \in [0, 1]} \mathbb{E} \{q C_{\text{RF}}^I(H_1)\},
\]

\[
\mathbb{E} \{q (1-q) C_{\text{RF}}^I(H_2)\} + M \mathbb{E} \{C_{\text{FSO}}(g)\}. \tag{24}
\]

The solution to the above optimization problem is given in the following proposition.

Proposition 1: The optimal DT-FL policy for the considered cascaded RF and hybrid RF/FSO system is given by

\[
q^* = \begin{cases} \frac{M \mathbb{E} \{C_{\text{FSO}}(g)\} - \mathbb{E} \{C_{\text{RF}}^I(H_2)\}}{\mathbb{E} \{C_{\text{RF}}^I(H_1)\} + \mathbb{E} \{C_{\text{FSO}}(g)\}}, & \text{if } \mathbb{E} \{C_{\text{RF}}^I(H_1)\} > M \mathbb{E} \{C_{\text{FSO}}(g)\} \tag{25} \\
1, & \text{otherwise} \end{cases}
\]

The achievable sum throughput of the above optimal DT-FL policy is given by

\[
\tau^* = q^* \mathbb{E} \{C_{\text{RF}}^I(H_1)\}. \tag{26}
\]

Proof: The two terms in the min function in (24) are linear and monotonic in \(q\). Hence, the solution is either at their intersection or at the boundaries. If \(\mathbb{E} \{C_{\text{RF}}^I(H_1)\} > M \mathbb{E} \{C_{\text{FSO}}(g)\}\) holds, the two arguments in the min function become identical and the solution is the boundary point \(q^* = 1\) since the other boundary point \(q = 0\) leads to \(\tau = 0\). The optimal throughput is the data rate of the users as given in (26) since all the data received from the users at the relay will be forwarded to the destination. This completes the proof.

2) Delay-Limited Transmission: Unlike the optimization problem in (21) for the DL-AL policy, the optimization problem for the DL-FL policy cannot be equivalently stated as the independent optimization of the policy in each transmission block. In fact, in contrast to the DL-AL policy, the optimal throughput \([b] = q^*\) in the DL-FL policy is fixed for all transmission blocks, and hence, the optimal \(q^*\) has to be found such that the throughput averaged over all transmission blocks is maximized. In particular, the optimization problem for the DL-FL policy has to be formulated as follows

\[
\text{maximize } \tau = N \mathbb{E} \{\min_{q \in [0, 1]} (1-q) \mathbb{E} \{q C_{\text{RF}}^I(H_1)\}, (1-q) \mathbb{E} \{C_{\text{RF}}^I(H_2)\} + M \mathbb{E} \{C_{\text{FSO}}(g)\}\}. \tag{27}
\]

In the following proposition, the optimal solution of (27) is provided.

Proposition 2: The optimal DL-FL policy for the considered cascaded RF and hybrid RF/FSO system is given by

\[
q^* = \begin{cases} q_{\text{sol}}^*, & \text{if } \mathbb{E} \{C_{\text{RF}}^I(H_1)\} > M \mathbb{E} \{C_{\text{FSO}}(g)\} \tag{28} \\
1, & \text{otherwise} \end{cases}
\]

where \(q_{\text{sol}}^*\) is numerically obtained to satisfy the following integral equation

\[
\int_{x=0}^{\infty} \int_{y=0}^{\infty} \frac{(1-q)((1-q)y + z)}{q^2} f_x(x) f_y(y) f_z(z) dy dx = 0, \tag{29}
\]

where \(x = NC_{\text{RF}}^I(H_1), y = NC_{\text{RF}}^I(H_2),\) and \(z = NMC_{\text{FSO}}(g)\). Moreover, \(f_x(x), f_y(y),\) and \(f_z(z)\) are the probability density functions of \(x, y,\) and \(z,\) respectively. The achievable throughput of the optimal DL-FL policy is given by

\[
\tau = \int_{x=0}^{\infty} \int_{y=0}^{\infty} \left(\frac{1-q}{q^2} + \frac{1}{q^2}\right) f_x(x) f_y(y) f_z(z) dy dz. \tag{30}
\]

Proof: The two terms in the min function in (27) are monotonic functions in \(q\), hence, there exists a unique solution for \(q \in [0, 1]\). The optimal \(q^*\) can be obtained by setting the derivative of \(\tau\) equal to zero. Using the general form of the Leibniz integral rule [36], \(\frac{d\tau}{dq}\) is obtained as in (29). Note that the \(q^*\) obtained from (29) is valid only if it belongs to \(q \in [0, 1]\). In order to verify this, we have to check the boundary points \(q = 0, 1\). It is clear that \(q = 0\) cannot be the optimal solution as it leads to \(\tau = 0\). Moreover, substituting \(q = 1\) into (29), we obtain \(\mathbb{E} \{C_{\text{RF}}^I(H_1)\} = M \mathbb{E} \{C_{\text{FSO}}(g)\}\) holds. Hence, if \(\mathbb{E} \{C_{\text{RF}}^I(H_1)\} > M \mathbb{E} \{C_{\text{FSO}}(g)\}\) holds, we obtain \(q^* = q_{\text{sol}}^* < 1\). Otherwise, if \(\mathbb{E} \{C_{\text{RF}}^I(H_1)\} \leq M \mathbb{E} \{C_{\text{FSO}}(g)\}\) holds, we obtain \(q_{\text{sol}}^* \geq 1\) and hence \(q^* = 1\). This result is formally stated in (28). Moreover, for a given \(q^*\), the throughput in (27) can be written as \(\tau = \min\{q^* x, (1-q^*) y + z\}\), where, for given \(y, z\), we obtain \(\tau = q^* x\) if \(x \leq \frac{(1-q^*) y}{q^*} + \frac{z}{q^*}\) holds and \(\tau = (1-q^*) y + z\) if \(x \geq \frac{(1-q^*) y}{q^*} + \frac{z}{q^*}\) holds. This result is given in integral form in (30). This completes the proof.

Remark 3: By comparing the optimal DT-AL, DL-AL, DT-FL, and DL-FL policies in Theorem 1 (see also Corollary 1), Theorem 2, Proposition 1, and Proposition 2, respectively, we can conclude that the threshold for activating the \(\mathcal{R-D}\) RF link and not activating it at all is identical for the optimal DT-AL, DT-FL, and DL-FL policies. In particular, if \(\mathbb{E} \{C_{\text{RF}}^I(H_1)\} > M \mathbb{E} \{C_{\text{FSO}}(g)\}\) holds, the \(\mathcal{R-D}\) RF link is activated, otherwise, it remains inactive for all transmission blocks. In contrast, the \(\mathcal{R-D}\) RF link cannot remain inactive for all transmission blocks for the optimal DL-AL policy.

Remark 4: We note that the proposed problem formulation and the resulting protocols in Theorems 1 and 2 and Propositions 1 and 2 are provided in a general form such that coding schemes differ from those considered in this paper, cf. Section III.B, can be easily accommodated. In particular, using different coding schemes changes only the values of
the instantaneous capacity expressions in (9), (10), and (12) such that the general results, i.e., the proposed link allocation policies, remain valid.

V. NUMERICAL RESULTS

Unless otherwise stated, the values of the parameters for the RF and FSO links used for the numerical results shown in this section are provided in Table II. Moreover, we assume that for each transmission block, the users are randomly distributed within an area that is defined by two concentric circles having radii of $d_{\text{max}} = 600$ m and $d_{\text{min}} = 100$ m, respectively, and centered at the origin. The relay and destination nodes are fixed where the relay is located at the origin and the destination is located at a distance $d_{\text{RD}}$ from the origin. Furthermore, we consider the following two protocols as benchmark schemes i) mixed RF/FSO relaying which does not employ an RF backup link [15] and ii) conventional RF relaying [38] which does not employ an FSO backhaul link. Thereby, in mixed RF/FSO relaying, the relay forwards the data received in each block from the users to the destination in the following block over the FSO link. In contrast, in conventional RF relaying, the relay receives data from the users in one transmission block and forwards this data to the destination in the next transmission block. The achievable throughputs of mixed RF/FSO relaying and conventional RF relaying are denoted by $\tau^{\text{mix}}$ and $\tau^{\text{conv}}$, respectively, and are given by

$$\tau^{\text{mix}} = N \times \mathbb{E} \{ \min \{ C_{\text{RF}}(H_1), M C_{\text{FSO}}(g) \} \}$$

and

$$\tau^{\text{conv}} = \frac{N}{2} \times \mathbb{E} \{ \min \{ C_{\text{RF}}(H_1), C_{\text{RF}}^{\text{conv}}(H_2) \} \}$$

respectively. We assume that the relay employs a ZF detector to recover the information sent by the users for all considered protocols.

In Fig. 3, we show the normalized achievable sum throughput of all users $\bar{\tau} = \frac{\tau}{T_{\text{UR}}} \cdot \frac{1}{\text{bit} / \text{s} / \text{Hz}}$ versus the weather-dependent attenuation coefficient of the FSO link, $\kappa$, for $K = J = L = 2$, $d_{\text{RD}} = 1000$ m, and $R_{Uk} = 4$ bits/symbol, $\forall k$. From Fig. 3, we observe that for favorable atmospheric conditions, i.e., low values of $\kappa$, the relay is able to forward all the information received from the users to the destination over the FSO link. Hence, for the DT-AL, DT-FL, and DL-FL policies, the $\mathcal{R}$-$\mathcal{D}$ RF backhaul link remains inactive for all transmission blocks, see Remark 3. As a result, the achievable throughput of the proposed policies and mixed RF/FSO relaying coincide. However, as $\kappa$ increases, i.e., the quality of the FSO link deteriorates due to severe atmospheric conditions, there is a critical value of $\kappa$ above which the FSO link is no longer able to support all user data. Hence, the achievable throughput of the mixed RF/FSO relaying protocol decreases ultimately to zero for large values of $\kappa$. In contrast, although the achievable throughputs of the proposed policies also decrease due to the sharing of the transmission time between the $\mathcal{U}$-$\mathcal{R}$ and $\mathcal{R}$-$\mathcal{D}$ RF links, they converge to non-zero values for large values of $\kappa$. The achievable throughput of the conventional RF relaying protocol does not depend on the quality of the FSO link. Moreover, for high values of $\kappa$ when the FSO link becomes unavailable, the proposed protocols still outperform the conventional RF relaying protocol due to the optimized adaptive/fixed link allocation. In fact, for $K \rightarrow \infty$, i.e., when the FSO link becomes unavailable, the proposed protocols converge to the corresponding optimal RF relaying protocols, see e.g. [20], [21]. Finally, as can be observed from Fig. 3, the DT-AL and DL-FL policies have the highest and lowest performance among the proposed policies, respectively. Moreover, the throughputs of the DL-AL and DT-FL policies fall between those of the DT-AL and DL-FL policies.
We assume $\tau^{\text{RD}} = 2000$ m. From low to high values of $\kappa$, the vertical dashed-dotted lines represent the following weather conditions [8]: clear air, haze, light fog, and moderate fog, respectively.

In Fig. 4, we consider the same schemes as in Fig. 3 for the same parameters except that we now assume $d_{RF}^{\text{RD}} = 2000$ m. Similar observations as in Fig. 3 can be made for Fig. 4. However, by comparing Figs. 3 and 4, we can observe that the critical value of $\kappa$ above which transmission time sharing between the RF links becomes necessary, depends on the system parameters. For instance, for the parameters considered here and moderate foggy atmospheric conditions, i.e., $\kappa = 20 \times 10^{-3}$, the $\text{RF}$ link is active if $d_{RF}^{\text{RD}} = 2000$ m but inactive if $d_{RF}^{\text{RD}} = 1000$ m. Moreover, for large values of $\kappa$, we observe that the performance gain of the proposed DL-FL policy compared to conventional RF relaying in Fig. 4 is smaller than in Fig. 3. The reason is that for $d_{RF}^{\text{RD}} = 2000$, the average capacities of the $\text{U-R}$ and the $\text{RF}$ links have similar values. Hence, the optimal value of $q^*$ obtained from Proposition 2 is close to $\frac{1}{2}$ which leads to similar performances of the proposed DL-FL policy and the conventional RF relaying protocol.

In Figs. 5 and 6, we show the normalized achievable sum throughput of all users, $\bar{\tau}$ (in Mbits/second), versus the transmit power of the users, $P_{U_k}^{RF}$ (in dBm), for $K = J = L = 2$, $d_{RF}^{\text{RD}} = 1000$ m, $P_{U_k}^{RF} = P_{U_k}^{RF} + 10$ dBm [37], and $R_{U_k} = 4$ bits/symbol $\forall k$. Moreover, the weather-dependent attenuation coefficient of the FSO link is set to $\kappa = 34 \times 10^{-3}$ and $\kappa = 36 \times 10^{-3}$ in Figs. 5 and 6, respectively. We observe from these figures that the average throughputs of all protocols increase as the transmit power of the users increases. Nevertheless, we note that since the users transmit with fixed transmission rate, the throughputs of the proposed policies, conventional RF relaying, and mixed RF/FSO relaying ultimately saturate at $\bar{\tau} = KR_{U_k}^{RF} = 160$ Mbits/second, $\bar{\tau} = 0.5R_{U_k}^{RF} = 80$ Mbits/second, and $\bar{\tau} = \mathbb{E}\{\min\{KR_{U_k}^{RF}, C_{FSO}(r)\}^{W_{FSO}}\} = 105, 44$ Mbits/second, respectively, as $P_{U_k}^{RF} \to \infty$. Furthermore, the proposed policies outperform mixed RF/FSO relaying and conventional RF relaying by a considerable margin. For the set of parameters considered here, mixed RF/FSO relaying outperforms conventional RF relaying in Fig. 5 for $\kappa = 34 \times 10^{-3}$ but is outperformed by conventional RF relaying in Fig. 6 for $\kappa = 36 \times 10^{-3}$.

In Fig. 7, we show the normalized achievable sum throughput of all users, $\bar{\tau}$ (in Mbits/second), versus the transmit power of the users, $P_{U_k}^{RF}$, for $\kappa = 36 \times 10^{-3}$. The horizontal dashed-dotted lines denote asymptotic limits as $P_{U_k}^{RF} \to \infty$. respectively, as $P_{U_k}^{RF} \to \infty$. Furthermore, the proposed policies outperform mixed RF/FSO relaying and conventional RF relaying by a considerable margin. For the set of parameters considered here, mixed RF/FSO relaying outperforms conventional RF relaying in Fig. 5 for $\kappa = 34 \times 10^{-3}$ but is outperformed by conventional RF relaying in Fig. 6 for $\kappa = 36 \times 10^{-3}$.
sufficiently good such that the relay can forward all data received from the users to the destination only via the FSO link. However, as $K$ increases, the relay requires also the RF-RF link to forward data received from the users to the destination. Furthermore, as $K \to \infty$, the throughputs of the proposed policies as well as the conventional relaying protocol are limited by the quality of the FSO link and saturates at $\bar{\tau} = \mathbb{E}\{C_{FSO}(g)W_{FSO}\} = 240$ Mbits/second.

VI. CONCLUSIONS AND FUTURE WORK

A cascaded RF and hybrid RF/FSO system was considered where multiple users transmit their data over an RF link to a relay node and the relay forwards the information to the destination over a hybrid RF/FSO backhaul link. The optimal fixed and adaptive link allocation policies, which assign the transmission time to the RF links, were derived for both delay-limited and delay-tolerant transmission. In particular, the four proposed link allocation policies, namely DT-AL, DL-AL, DT-FL, and DL-FL, maximize the achievable throughput for given delay requirements and signaling overheads. Moreover, we could show that for the four considered scenarios, transmission time sharing becomes necessary for throughput maximization if the instantaneous and/or statistical quality of the FSO link falls below a certain threshold. Our numerical results revealed the effectiveness of the proposed system architecture and link allocation policies even in cases when the FSO link is affected by adverse atmospheric conditions. An interesting topic for future work is the extension of the presented link allocation schemes to other network architectures with hybrid RF/FSO links, such as the multi-hop serial/two-hop parallel relaying architectures [39], [40].

APPENDIX A
PROOF OF THEOREM 1

In this appendix, our aim is to find the optimal link allocation policy as a solution of the optimization problem given in (15). Since the cost function and the constraints in (15) are affine in the optimization variables $q[b]$, $\theta[b]$, and $\tau^{upp}$ and the feasible set is non-empty, Slater’s condition is satisfied. Hence, the duality gap is zero [41]. Therefore, the solution of the primal problem in (15) can be found from the solution of the dual problem of (15) [42]. Denoting the Lagrange multipliers corresponding to constraints C1 and C2 by $\omega_1$ and $\omega_2$, respectively, the Lagrangian function corresponding to the optimization problem in (15) is obtained as

$$L(q[b], \tau^{upp}, \omega_1, \omega_2) = \tau^{upp} + \omega_1 \left[ \frac{1}{B} \sum_{b=1}^{B} q[b] N_C^R [b] - \tau^{upp} \right] + \omega_2 \left[ \frac{1}{B} \sum_{b=1}^{B} \left(1 - q[b]\right) N_C^F [b] + M N_C^{FSO} [b] - \tau^{upp} \right].$$

The dual function is then given by

$$\mathcal{D}(\omega_1, \omega_2) = \max_{q[b] \in [0,1], \tau^{upp} \geq 0} L(q[b], \tau^{upp}, \omega_1, \omega_2) \quad (34)$$

and the dual problem is given by

$$\min_{\omega_1 \geq 0, \omega_2 \geq 0} \mathcal{D}(\omega_1, \omega_2). \quad (35)$$

To solve (15) using the dual problem in (35), we first obtain the primal variables $q[b]$ and $\tau^{upp}$ for given dual variables $\omega_1$ and $\omega_2$ from (34). Then, the optimal $\omega_1$ and $\omega_2$ are obtained by solving the dual problem in (35).

A. Optimal Primal Variables

The optimal link allocation variables, $q[b], \forall b$, and the optimal value of the upper bound, $\tau^{upp}$, are either the boundary points of the feasible sets, i.e., $q[b] \in [0,1]$ and $\tau^{upp} \geq 0$, or the stationary points which can be obtained by setting the derivatives of the Lagrangian function in (33) with respect to $\tau^{upp}$ and $q[b]$ to zero. The derivatives of the Lagrangian function in (33) are given by

$$\frac{\partial L}{\partial \tau^{upp}} = 1 - \omega_1 - \omega_2 \quad (36a)$$

and

$$\frac{\partial L}{\partial q[b]} = \frac{1}{B} \left[ \omega_1 N_C^R [b] - \omega_2 N_C^R [b] \right]. \quad (36b)$$

If the derivative $\frac{\partial L}{\partial \tau^{upp}}$ is non-zero, the optimal value of $\tau^{upp}$ is at the boundary of its feasible set, i.e., $\tau^{upp} \to \infty$ or $\tau^{upp} \to 0$, which cannot be the optimal solution. Therefore, the derivative $\frac{\partial L}{\partial q[b]}$ in (36a) has to be zero which leads to $\omega_1 + \omega_2 = 1$. Since the probability density functions of the channel coefficients $H_1$ and $H_2$ are continuous, we obtain that $\Pr \left\{ \frac{\partial L}{\partial q[b]} = 0 \right\} = 0$ holds. Hence, the optimal value of $q[b]$ is always at the boundaries, i.e.,

$$q^*[b] = \begin{cases} 0, & \text{if } \omega_1 C_1^R [b] < \omega_2 C_2^R [b] \\ 1, & \text{otherwise} \end{cases} \quad (37)$$
We note that for a given \( \omega \) of function in (34). The dual function \( q \) problem in (35). In particular, substituting, the optimal values of \( B \). Optimal Dual Variable to hold. To find the optimal dual variable, we can employ as can be expressed independent from the dynamics of the queue as

\[
\frac{\partial D(\omega)}{\partial \omega} = \frac{1}{B} \sum_{b=1}^{B} [q^* b - q_{C_1}^{RF} b + q_{C_2}^{FSO} b + MNC_{FSO} b].
\]

We note that for a given \( \omega \), the value of \( \frac{\partial D(\omega)}{\partial \omega} \) is fixed. Thereby, we can investigate the following mutually exclusive cases for the optimal \( \omega^* \):

Case 1: \( \omega^* > 1 \) and \( \omega^* < 0 \) cannot hold due to dual feasibility condition [41]. In particular, the Lagrange multipliers corresponding to inequality constraints must be non-negative. Hence, we obtain \( \omega_1 \geq 0 \) and \( \omega_2 \geq 0 \) which leads to \( 0 \leq \omega \leq 1 \).

Case 2: If we assume that \( \omega^* = 0 \) holds, we obtain \( q[b] = 0, \forall b \) from (37). Intuitively, this cannot be the optimal dual variable since it leads to \( \tau^{\text{upp}} > 0 \). In particular, the derivative \( \frac{\partial D(\omega)}{\partial \omega} \) becomes negative and in order to minimize \( D(\omega) \), we have to increase \( \omega \). In fact, we can assume a small \( \omega = \epsilon \) and obtain a positive \( \tau^{\text{upp}} \) and decrease the dual function. Hence, \( \omega^* = 0 \) cannot hold for the optimal solution.

Case 3: Considering Case 1 and Case 2, \( 0 < \omega^* \leq 1 \) has to hold. To find the optimal dual variable, we can employ the widely-adopted gradient method [43]. The main idea is to minimize \( D(\omega) \) by updating \( \omega \) along the gradient search direction. The updates are performed as

\[
\omega[i + 1] = \left[ \omega[i] - \delta[i] \frac{\partial D(\omega)}{\partial \omega} \right]_{\omega = \omega[i]}. \tag{39}
\]

This leads to the iterative algorithm in Theorem 1 with the updates given in (17). The gradient method is guaranteed to converge to the optimal dual variable \( \omega^* \) provided that the step sizes \( \delta[i] \) are chosen sufficiently small [43]. This completes the proof.

APPENDIX B
PROOF OF LEMMA 1

The proof follows from a well-known result in queuing theory [44], [45]. Let us consider a buffer with stochastic arrival process \( A[b] \), queue capacity process \( C[b] \), and queue length \( Q[b] \) in the \( b \)-th time instance. Thereby, the departure rate is obtained as \( D[b] = \min(Q[b - 1], C[b]) \) and \( Q[b] \) evolves as \( Q[b] = \max(Q[b - 1] - C[b], 0) + A[b] \). However, the average departure rate \( E[D] \) (averaged over the time blocks) can be expressed independent from the dynamics of the queue as

\[
E[D] = \begin{cases} E(C), & \text{if } E(A) > E(C) \\ E(A) = E(C), & \text{if } E(A) = E(C) \\ E(A), & \text{if } E(A) < E(C) \end{cases} \tag{40}
\]

In particular, if \( E(A) > E(C) \) holds, i.e., the average rate flowing into the buffer is larger than the average capacity of the respective departure channel, then, as the number of time instances grow to infinity, the buffer always has enough information to supply because the amount of information in the queue increases over time and we obtain \( E[D] = E(C) \). On the other hand, if \( E(A) < E(C) \) holds, i.e., the average information flowing into the buffer is less than the average capacity of the respective departure channel, then, by the law of conservation of flow, we obtain \( E[D] = E(A) \). If \( E(A) = E(D) \) holds, we obtain \( E[D] = E(A) = E(C) \) due to the continuity property. Hence, from (40), we can conclude that \( E[D] = \min(E(A), E(C)) \) holds. Considering the queue at the relay node, \( NC_{C_1}^{RF}[b] + NC_{C_2}^{FSO}[b] + MNC_{FSO} b \) in our system correspond to \( A \) and \( C \), respectively. Hence, the average throughput is given by \( \tau = E[D] = \min(NC_{C_1}^{RF}, NC_{C_2}^{FSO}) \) which is identical to the upper bound in (19), i.e., \( \tau = \tau^{\text{upp}} \). This completes the proof.

APPENDIX C
PROOF OF COROLLARY 1

If \( E[C_{FSO}^{RF}(H_1)] \leq M \), \( E[C_{FSO}^{FSO}(g)] \) holds, from (38), we obtain that \( \frac{\partial D(\omega)}{\partial \omega} < 0 \) holds regardless of the value of \( \omega \). Hence, in order to minimize the dual function \( D(\omega) \), we have to increase \( \omega \). Moreover, since \( \omega \in (0, 1) \) holds, we obtain \( \omega^* = 1 \). On the other hand, if \( E[C_{C_1}^{RF}(H_1)] > M \), \( E[C_{FSO}^{FSO}(g)] \) holds, we can conclude that \( 0 < \omega^* < 1 \) and its value is unique. In other words, according to (19), \( \tau^{\text{upp}} = N \min(C_{C_1}^{RF}, C_{2}^{FSO}, C_{FSO}) \) where \( C_{C_1}^{RF} \) is a monotonically increasing function of \( \omega \), \( C_{2}^{FSO} \) is a monotonically decreasing function of \( \omega \), and \( C_{FSO} \) does not depend on \( \omega \). Therefore, the optimal \( \omega \) has to be chosen such that \( C_{1}^{RF} = C_{2}^{RF} + C_{FSO} \) holds which leads to (20). This completes the proof.

ACKNOWLEDGMENT

This publication was made possible by the NPRP award [NPRP 5-157-2-051] from the Qatar National Research Fund (a member of the Qatar Foundation). The statements made herein are solely the responsibility of the authors.

REFERENCES

Diomidis S. Michalopoulos (S’05-M’10-SM’15) was born in Thessaloniki, Greece, in 1983. He received the Diploma in engineering (5 year studies) and the Ph.D. degree from the Electrical and Computer Engineering Department, Aristotle University of Thessaloniki, in 2005 and 2009, respectively. In 2009, he joined the University of British Columbia, Canada, as a Killam postdoctoral fellow, and in 2011, he was awarded a Banting postdoctoral fellowship. From 2014 to 2015 he was with the University of Erlangen-Nuremberg, Germany, as researcher and teaching instructor. Since 2015 he is with Nokia Networks, Germany, as Radio Systems Research Engineer. His research interests span the broad area of digital wireless communications, with emphasis on physical layer as well as radio access aspects. Dr. Michalopoulos received the Marconi Young Scholar award from the Marconi Society in 2010, the Killam postdoctoral fellow research prize for excellence in research in the University of British Columbia in 2011, and the Best Paper Award of the Wireless Communications Symposium (WCS) in the IEEE International Conference on Communications (ICC) 2007. He has served as an Associate Editor of the IEEE COMMUNICATIONS LETTERS (2010-2015), as well as member of Technical Program Committees for major IEEE conferences such as Globecom, WCNC, and VTC.

Murat Uysal (S’98-M’02-SM’07) is currently a Full Professor and Chair of the Department of Electrical and Electronics Engineering at Ozyegin University, Istanbul, Turkey. Prior to joining Ozyegin University, he was a tenured Associate Professor at the University of Waterloo, Canada, where he still holds an adjunct faculty position. Dr. Uysal’s research interests are in the broad areas of communication theory and signal processing with a particular emphasis on the physical layer aspects of wireless communication systems in radio, acoustic and optical frequency bands. He has authored some 250 journal and conference papers on these topics and received more than 5000 citations.

Prof. Uysal currently serves as the Chair of IEEE Turkey Section. He leads the EU COST Action OPTICWISE which is a European scientific network for interdisciplinary research activities in the area of optical wireless communications. He serves on the editorial boards of IEEE TRANSACTIONS ON COMMUNICATIONS and IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. In the past, he served as an Editor for IEEE COMMUNICATIONS LETTERS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, Wiley Wireless Communications and Mobile Computing (WCMC) Journal, Wiley Transactions on Emerging Telecommunications Technologies (ETT), and Guest Editor of IEEE JSAC SPECIAL ISSUES ON OPTICAL WIRELESS COMMUNICATION (2009 and 2015). He was involved in the organization of several IEEE conferences at various levels. He served as the Chair of the Communication Theory Symposium of IEEE ICC 2007, Chair of the Communications and Networking Symposium of IEEE CCECE 2008, Chair of the Communication and Information Theory Symposium of IWCMC 2011, TPC Co-Chair of IEEE IWOW 2012, TPC Co-Chair of IEEE WCNC 2014 and General Chair of IEEE IWOW 2015. His distinctions include the Marsland Faculty Fellowship in 2004, NSERC Discovery Accelerator Supplement Award in 2008, University of Waterloo Engineering Research Excellence Award in 2010, Turkish Academy of Sciences Distinguished Young Scientist Award in 2011 and Ozyegin University Best Researcher Award in 2014 among others.

Robert Schober (S’98, M’01, SM’08, F’10) was born in Neuendettelsau, Germany, in 1971. He received the Diplom (Univ.) and the Ph.D. degrees in electrical engineering from the University of Erlangen-Nuremberg in 1997 and 2000, respectively. From May 2001 to April 2002 he was a Postdoctoral Fellow at the University of Toronto, Canada, sponsored by the German Academic Exchange Service (DAAD). Since May 2002 he has been with the University of British Columbia (UBC), Vancouver, Canada, where he is now a Full Professor. Since January 2012 he is an Alexander von Humboldt Professor and the Chair for Digital Communication at the Friedrich Alexander University (FAU), Erlangen, Germany. His research interests fall into the broad areas of Communication Theory, Wireless Communications, and Statistical Signal Processing.

Dr. Schober received several awards for his work including the 2002 Heinz Maier-Leibnitz Award of the German Science Foundation (DFG), the 2004 Innovations Award of the Vodafone Foundation for Research in Mobile Communications, the 2006 UBC Killam Research Prize, the 2007 Wilhelm Friedrich Bessel Research Award of the Alexander von Humboldt Foundation, the 2008 Charles McDowell Award for Excellence in Research from UBC, a 2011 Alexander von Humboldt Professorship, and a 2012 NSERC E.W.R. Steacie Fellowship. In addition, he received best paper awards from the German Information Technology Society (ITG), the European Association for Signal, Speech and Image Processing (EURASIP), IEEE WCNC 2012, IEEE Globecom 2011, IEEE ICUWB 2006, the International Zurich Seminar on Broadband Communications, and European Wireless 2000. Dr. Schober is a Fellow of the Canadian Academy of Engineering and a Fellow of the Engineering Institute of Canada. From 2012 to 2015 he served as Editor-in-Chief of the IEEE TRANSACTIONS ON COMMUNICATIONS. He is currently the Chair of the Steering Committee of the IEEE TRANSACTIONS ON MOLECULAR, BIOLOGICAL AND MULTISCALE COMMUNICATION and a Member-at-Large on the Board of Governors of the IEEE Communication Society.